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METHOD OF SUCCESSIVE APPROXIMATIONS FOR THE INTEGRATION OF
EQUATIONS OF A LAMINAR MULTI-COMPONENT BOUNDARY LAYER
WITH CHEMICAL REACTIONS INCLUDING IONIZATION

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Translation of "Metod posledovatel'nykh priblizheniy dlya integrirvaniya uravneniy laminarnogo mnogokomponentnogo pogranichnogo sloya s khimicheskimi reaktsiyami vklyuchaya reaktsii ionizatsii". Paper presented at the 21st International Astronautical Congress, October 4 - 10, 1970, Constance, West Germany, Moscow, Moscow State University, Institute of Mechanics, Report, December, 1969, 52 pages.

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METHOD OF SUCCESSIVE APPROXIMATIONS FOR THE INTEGRATION OF
EQUATIONS OF A LAMINAR MULTI-COMPONENT BOUNDARY LAYER
WITH CHEMICAL REACTIONS INCLUDING IONIZATION

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ABSTRACT. The two-dimensional stationary boundary layer equations of an asymptotically thin boundary layer are solved using the method of successive approximation. A multi-component gas with chemical reactions and ionization is considered. The method was tested and found to be accurate to 1%.

ANNOTATION

This work represents further development and application of the method of successive approximations [1] to the integration of two-dimensional stationary equations of an asymptotically thin laminar boundary layer in a precise formulation for the general case of the motion of a multi-component chemically reacting gas, including ionization.

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One of the advantages of this method of successive approximations lies particularly in the fact that the first approximations may be computed even for complex problems of a hypersonic boundary layer in analytical form, and will yield a good approximate solution, whose accuracy may be computed using this method by computing the successive approximations on an electronic computer.

The convergence of this method was experimentally proven on simple prob-

* Numbers in the margin indicate the pagination in the original foreign text.

lems of boundary layer theory by computing a large number of iterations (up to 20), which showed that with a reasonable assignment of the zero approximation the third to fourth approximations will yield a solution that differs no more than 1% from the exact one.

The numerical results of the systematic computations according to this method will be cited in a subsequent paper.

We have simultaneously given a strict rational formulation of the problem on a chemically balanced boundary layer on an impermeable and thermochemically disintegrated* wall, when the diffusion and thermodiffusion properties of the components are different.

The basic equations for a partially ionized boundary layer without external electromagnetic fields with reactions are reduced to a type which agree in form with the equations for a chemically reacting mixture of neutral gases.

*Translator's note: The term ablation is obviously designated here within the context of re-entry physics being discussed. The author uses a different term, however, and the translation follows his nomenclature and uses the word disintegration, instead of ablation.

INTRODUCTION

Different methods are used at the present time as the basic methods for obtaining "precise" solutions to the boundary layer equations. The method of integral relationships proposed by A. A. Dorodnitsin [2] has also received wide-spread distribution in the past few years.

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The use itself and the application of one of these methods to complex physico-chemical boundary layers in the presence of a large number of components, such as diffusion, unbalanced homogeneous and heterogeneous chemical reactions and ionization reactions is always an independent and rather difficult problem which can often not be solved even on modern electronic computers.

Thus, on the one hand, the existence of complicated mathematical problems in boundary layer theory, and on the other hand, the rather complex methods for obtaining their precise solutions leads to the fact that in the operation of institutes of design and departments of design serial computations of boundary layers are carried out with correlation formulas, obtained either on the basis of numerical computations of problems in a simplified physical formulation or on the basis of approximate methods based primarily on the integral method of Karman-Pohlhausen.

Significant advances in the creation of approximate methods in boundary layer theory were attained in recent years in connection with the appearance of the parameteric method of integrating universal equations of the laminar boundary layer proposed by L. G. Loytsyanskiy [3].

With respect to the above in boundary layer theory, especially the hypersonic one, there is a necessity for creating sufficiently simple and reliable methods of computation.

In this paper we propose a new variation of the numerical method for obtaining "exact" solutions to the boundary layer equations, i.e., the method of successive approximations.

To some degree this method utilizes the method of analytical procedures compiled in the many years that boundary layer theory has been in existence combined with the numerical methods; and in this manner it permits us to obtain visible results even in the complex problems of boundary layer theory. The possibility of obtaining first approximations in analytical form from this method (as a rule, no more than two in complex problems; the zero approximation is given) permits us to write the approximate solutions in explicit form, the accuracy of which may be estimated using the method itself by computing the remote approximations on an electronic computer which will simultaneously give the "precise" solution to the problem as well.

At the present time the convergence of this method has not been proven. There is "experimental" evidence of the convergence of the method obtained by direct computation on an electronic computer of a large number of approximations (more than 20). Furthermore, the convergence of the method has been evidenced on the simplest model linear problems when the solution can be written in the common recurrent form at each step of the iterations.

A rational formulation of the problem is very significant in the application of any method to the complex problems of boundary layer theory. Therefore, much attention must be paid to this side of the question before the method of successive approximations is used. First of all the equations for a partially ionized boundary layer with the respective exclusion of an electric field arising due to separation of the charged components during diffusion are reduced to the type which agrees in form with the ordinary boundary layer equations for a mixture of neutral components. Secondly, by using the conditions of chemical balance, the equations for a chemically balanced boundary layer are reduced to a new form suitable for actual solution.

We shall discuss in detail the boundary conditions both on an impermeable surface and on a thermochemical wall. We shall cite the exact structural formula for the effective enthalpy of the thermochemical disintegration of the wall material of arbitrary chemical composition.

§ 1. METHOD OF SUCCESSIVE APPROXIMATIONS

Let us discuss, using the method of successive approximations [1], the simplest problem of integrating the boundary layer equations for the case of an incompressible liquid. For sufficiently large Reynolds numbers the flow of a viscous liquid near a body is described by a Prandtl system of equations [4]

$$\begin{aligned} \frac{\partial}{\partial x}(u'z^k) + \frac{\partial}{\partial y}(v'z^k) &= 0 \\ u' \frac{\partial u'}{\partial x} + v' \frac{\partial u'}{\partial y} &= u_e \frac{du_e}{dx} + \nu \frac{\partial^2 u'}{\partial y^2} \end{aligned} \quad (1.1)$$

Here $0 \leq x \leq X$, $0 \leq y \leq \infty$. In this case we must satisfy the conditions:

$$u'(x, 0) = 0, \quad v'(x, 0) = v_w(x), \quad u'(x, y) \rightarrow u_e(x) \quad \text{when } y \rightarrow \infty \quad (1.2)$$

To these conditions must be added the "initial" condition when $x = 0$. The initial profile is given in problems on extension of the boundary layer:

$$u'(0, y) = u_0(y), \quad u_0 > 0 \quad \text{when } y > 0 \quad (1.3)$$

and we must determine the flow in a boundary layer when $x > 0$.

In problems of plane-parallel symmetrical and axisymmetrical flows, we have a physically obvious condition on the center line of the current passing through the leading critical point

$$u'(0, y) = 0 \quad (1.4)$$

According to the physical sense of the problem, $u'(x,y) > 0$ when $x > 0$, $y > 0$.

In this problem x is the coordinate along the surface of the body, y is that along the normal to the surface of the body, u' , v' are projections of the velocity vector, respectively, along the axes x and y , $k = 0$ with flow around a two-dimensional profile, $k = 1$ with a flow around a body of revolution, $r(x)$ is the radius of the cross section of the body of revolution, ν is the coefficient of kinematic viscosity, $u_e(x)$ is the given velocity of the non-viscous flow on the body under consideration, $v_w(x)$ is a specified function which gives the flow law along the surface of a body of a liquid having the same properties as the oncoming flow. Reference [5] examines the questions on existence and uniqueness of the solution of this problem using the method of finite differences. Here we give the algorithm of the actual calculation of the solution to the problem and furthermore mention the method for obtaining the approximate solutions. Apparently the method for obtaining the solution may itself serve as the basis for evidence of the existence and uniqueness of the solution to the Problem (1.1) - (1.4). However, this question is not examined in the present paper.

Let us introduce the flow function $\psi(x,y)$

$$\frac{\partial \psi}{\partial x} = -v' r^k, \quad \frac{\partial \psi}{\partial y} = u' r^k$$

and transform it into new independent variables

$$x = x, \quad \eta = \frac{u_e r^k y}{\delta(x)} \quad (1.5)$$

where $\delta(x)$ is as yet an arbitrary function of x .

If we represent the flow function in the form

$$\psi(x,y) = \delta(x) f(x,\eta) \quad (1.6)$$

then the velocity components are described by it as:

$$\begin{aligned} u'(x, y) &= U_e(x)u(x, \eta), \quad u(x, \eta) = \frac{\partial f}{\partial \eta} \\ -v'z^k &= \delta'f + \delta \frac{\partial f}{\partial x} + \delta u \eta x \end{aligned} \quad (1.7)$$

and the momentum equation and the boundary conditions assume the following form

$$\begin{aligned} -(\delta^2 \frac{\partial f}{\partial x} + \delta \delta'f) \frac{\partial u}{\partial \eta} + \delta^2 u \frac{\partial u}{\partial x} &= \delta^2 (1-u^2) \frac{d \ln U_e}{dx} + \nu U_e z^{2k} \frac{\partial^2 u}{\partial \eta^2} \\ u(x, 0) &= 0 \quad (\delta'f)'_{\eta=0} = -v_w z^k \\ u(0, \eta) &= 0 \quad u(x, \infty) = 1 \end{aligned} \quad (1.8)$$

Problem (1.8) can be further simplified if we introduce the coordinate s instead of x

$$ds = \nu U_e(x) z^{2k}(x) dx \quad (1.9)$$

Then

$$\begin{aligned} \delta^2 u \frac{\partial u}{\partial s} - (\delta \delta'f + \delta^2 \frac{\partial f}{\partial s}) \frac{\partial u}{\partial \eta} &= \delta^2 (1-u^2) \frac{d \ln U_e}{ds} + \frac{\partial^2 u}{\partial \eta^2} \\ u(s, \eta) &= \frac{\partial f}{\partial \eta} \quad u(s, 0) = 0 \quad \nu U_e z^{2k} (\delta'f)'_{\eta=0} = -v_w z^k \\ u(0, \eta) &= 0 \quad u(s, \infty) = 1 \end{aligned} \quad (1.10) \quad \underline{15}$$

If we integrate the previous equation over the variable η from η to ∞ and use the known asymptotic function $u(s, \eta)$ and $f(s, \eta)$ at infinity [5]

$$u(s, \eta) \rightarrow 1, \quad \frac{\partial u}{\partial \eta} \rightarrow 0, \quad f(s, \eta) \rightarrow \eta - \psi(x), \quad (1-u)f \rightarrow 0 \quad \text{when } \eta \rightarrow \infty$$

we then find the following integro-differential equation

$$\frac{\partial u}{\partial \eta} = \delta' \delta [(1-u)\psi(s, \eta) + \Delta(u)] + \delta^2 [(1-u) \frac{\partial \psi}{\partial s} + \frac{\partial}{\partial s} \Delta(u)] + \delta^2 \beta(s) \Delta(u) + \delta(1-u) \psi_w' \quad (1.11)$$

where

$$\begin{aligned}
\varphi(s, \eta) &= \int_0^\eta u(s, \eta') d\eta' & \Delta(u) &= \int_0^\infty u(1-u) d\eta' \\
f(s, \eta) &= \varphi(s, \eta) + f_w(s) & \Delta(1) &= \int_0^\infty (1-u^2) d\eta' \\
\psi_w'(s) &= (\delta f_w)'_s = -\frac{v_w z^k}{\delta u_e z_2^k} & \beta(s) &= \frac{d\psi_w}{ds}
\end{aligned} \tag{1.12}$$

Incidentally let us mention that if we set $\delta = 1$ and $\eta = 0$ in (1.11) we then find the integral Karman relationship in Prandtl form in the variables s, η , on the basis of which the well-known approximate method of Karman-Pohlhausen has been developed [3].

However, we shall discuss further the exact method for the solution based on the use of the method of successive approximations.

For this let us integrate Equation (1.11) by η from 0 to η :

$$u(s, \eta) = A(s, \eta) \delta \delta' + [\beta(s) B(s, \eta) + C(s, \eta)] \delta^2 + D(s, \eta) \psi_w' \delta \tag{1.13}$$

$$A(s, \eta) = \int_0^\eta [(1-u)\varphi + \Delta(u)] d\eta' \quad B(s, \eta) = \int_0^\eta \Delta(1) d\eta'$$

$$C(s, \eta) = \int_0^\eta [(1-u) \frac{\partial \varphi}{\partial s} + \frac{\partial}{\partial s} \Delta(u)] d\eta' \quad D(s, \eta) = \int_0^\eta (1-u) d\eta' \tag{1.14}$$

Now let us define the function $\delta(s)$ with the specified function $u(s, \eta)$ as the solution to the following ordinary differential equation

$$\delta \delta' + \left[\beta(s) \frac{B(s, \infty)}{A(s, \infty)} + \frac{C(s, \infty)}{A(s, \infty)} \right] \delta^2 + \frac{D(s, \infty)}{A(s, \infty)} \psi_w' \delta = \frac{1}{A(s, \infty)} \tag{1.15}$$

which follows from Equation (1.13) if we direct η to infinity in it and use the boundary condition $u(s, \infty) = 1$. Now if we take any solution to Equation (1.15) and substitute in (1.13), we find an integro-differential equation of the type

$$u(s, \eta) = \phi(s, \eta, u)$$

where the operator ϕ will always satisfy the conditions

$$\phi(s, 0, 0) = 0 \quad \phi(s, \infty, 1) = 1 \quad (1.16)$$

Therefore, to obtain the solution, we can arrange the following iteration process

$$u^{(n+1)} = \phi(s, \eta; u^{(n)}) \quad (1.17)$$

which is characteristic in that at each step of the iteration the following boundary conditions will always be satisfied

$$\begin{aligned} u^{(n)}(s, 0) &= 0 \\ u^{(n)}(s, \infty) &= 1 \end{aligned} \quad (1.18)$$

The operator ϕ is easy to construct.

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In the absence of a flow the solution to Equation (1.15), which satisfies the condition $\delta(0) = 0$, will be the function

$$J^2(s) = \exp\left(-2 \int_0^s y ds'\right) \int_0^s 2A^{-1}(s', \infty) \exp\left(2 \int_0^{s'} y ds''\right) ds' \quad (1.19)$$

$$y(s) = \beta(s) \frac{B(s, \infty)}{A(s, \infty)} + \frac{C(s, \infty)}{A(s, \infty)} \quad (1.20)$$

Substitution of it into (1.13) will give the functional equation in the following explicit form

$$\begin{aligned} u(s, \eta) &= \phi(s, \eta; u) \\ \phi(s, \eta; u) &= \frac{A(s, \eta)}{A(s, \infty)} + \left\{ \beta(s) \left[B(s, \eta) - \frac{A(s, \eta)}{A(s, \infty)} B(s, \infty) \right] + C(s, \eta) \right\} \end{aligned} \quad (1.21)$$

where

$$- \frac{A(s, \eta)}{A(s, \infty)} C(s, \infty) \} \exp \left(-2 \int_0^s y ds' \right) \int_0^s \frac{A^{-1}(s', \infty)}{A(s', \infty)} \exp \left(2 \int_0^{s'} y ds'' \right) ds' \quad (1.22)$$

In the variables x, η the operator ϕ will be

$$\begin{aligned} \phi(x, \eta; u) = & \frac{A(x, \eta)}{A(x, \infty)} + \left\{ \beta(x) \left[B(x, \eta) - \frac{A(x, \eta)}{A(x, \infty)} B(x, \infty) \right] + C(x, \eta) - \right. \\ & \left. - \frac{A(x, \eta)}{A(x, \infty)} C(x, \infty) \right\} \exp \left(-2 \int_x^\infty y(x') dx' \right) \int_0^x \frac{\mu \ell_e^2 r^{2K}}{A(x', \infty)} \exp \left(2 \int_0^{x'} y(x'') dx'' \right) dx' \quad (1.22') \end{aligned}$$

where $\beta(x) = \frac{d \ln \mu \ell_e}{dx}$, the functions A, B, C, G are given by Expressions (1.14) and (1.20) in which s must be replaced by x .

The right-hand side of Equation (1.21), after substituting into it the arbitrary integrable function which satisfies only the conditions in (1.18), will always satisfy (1.16). Equation (1.21) may therefore serve as a basis for constructing the iterations according to the Algorithm (1.17) successively for each cross section $s = \text{const}$ ($x = \text{const}$), beginning from $s = x = 0$. If the process converges to a certain limiting function $\tilde{u}(s, y)$, then the solution to the original Problem (1.1) - (1.2) will be the expression

$$u'(x, y) = \mu \ell_e(x) \tilde{u}(s(x), y) \quad (1.23)$$

in which it is easy to verify direct substitution.

After making the necessary number of iterations, determined by the required accuracy, we can compute the friction on the wall by using Expression (1.17) which will give

$$\begin{aligned} \tau_w(s) = \left(\mu \frac{\partial u'}{\partial y} \right)_w = & \frac{\mu \ell_e^2 r^K}{\delta(s)} \left(\frac{\partial u}{\partial \eta} \right)_{\eta=0} = \mu \ell_e^2 r^K \left\{ \frac{\tilde{\delta}(u)}{A(s, \infty) \delta(s)} + \left[\beta(s) (\tilde{\delta}(u) - \tilde{\delta}(u)) \frac{B(s, \infty)}{A(s, \infty)} + \right. \right. \\ & \left. \left. + \frac{\partial}{\partial s} \tilde{\delta}(u) - \tilde{\delta}(u) \frac{C(s, \infty)}{A(s, \infty)} \right] \delta(s) \right\} \quad (1.24) \end{aligned}$$

or in the variable x

$$C_w(x) = \frac{\rho U_e}{2K} \left\{ \frac{\tilde{\sigma}(u)}{A(x, \infty)} \frac{U_e x^{2K}}{\tilde{\sigma}(x)} + \left[\beta(x) (\tilde{\sigma}(x) - \tilde{\sigma}(u)) \frac{B(x, \infty)}{A(x, \infty)} + \frac{\partial}{\partial x} \tilde{\sigma}(u) - \tilde{\sigma}(u) \frac{C(x, \infty)}{A(x, \infty)} \right] \tilde{\sigma}(x) \right\} \quad (1.25)$$

where

$$\tilde{\sigma}(u) = \int_0^u (1-u) d\eta, \quad \tilde{\sigma}(x) = \int_0^x (1-u^2) d\eta, \quad \beta(x) = \frac{d \ln U_e}{dx}$$

For self-similar solutions, i.e., when $U_e = C s^m$, $C = \text{const}$, we will have

$$A(s, \infty) \equiv A(\infty) = \text{const}, \quad u(s, \eta) \equiv u(\eta), \quad \eta = \frac{Kx}{s} \\ B(s, \infty) \equiv B(\infty) = \text{const}, \quad \beta(s) = \frac{m}{s}, \quad K = \frac{B(\infty)}{A(\infty)} \quad (1.26)$$

$$C(s, \infty) \equiv 0, \quad \tilde{\sigma}^2(s) = \frac{2s}{2m + B(\infty)}$$

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Then these solutions are obtained by iteration of the simpler equation

$$u(\eta) = \frac{A(\eta)}{A(\infty)} + \left[B(\eta) - \frac{A(\eta)}{A(\infty)} B(\infty) \right] \frac{2m}{2m + B(\infty)} \quad (1.27)$$

For example, the solution to the Blasius problem ($m = 0$) will be found completely from the simple functional equation

$$u(\eta) = \frac{A(\eta)}{A(\infty)}, \quad A(\eta) = \int_0^\eta [(1-u)\eta + \Delta(u)] d\eta' \quad (1.28)$$

In the presence of a flow ($\psi_w' \neq 0$) construction of the operator ϕ is more complex. In this case Equation (1.15) is an Abel equation of second kind

$$\sigma' \sigma' = f_2(s) \sigma^2 + f_1(s) \sigma + f_0(s) \\ f_0(s) = \frac{1}{A(s, \infty)}, \quad f_1(s) = -\frac{2(s, \infty) \psi_w'(s)}{A(s, \infty)}, \quad f_2(s) = -\beta(s) \frac{B(s, \infty)}{A(s, \infty)} - \frac{C(s, \infty)}{A(s, \infty)} \quad (1.29)$$

which by substitution of

$$\delta = [Z(s) + F_{12}(s)]E(s), \quad F_{12}(s) = \int f_2 \exp(-\int f_2 ds), \quad E = \exp(\int f ds)$$

is brought to the form

$$ZZ' = f_0 \exp(-2\int f_2 ds) - F_{12} Z' \quad (1.30)$$

With a flow equal to zero ($F_{12} = 0$), the solution to this equation will be the function $Z + \delta(s)E^{-1}(s)$, where $\delta(s)$ is given by Expression (1.19).

In the general case Equation (1.30) is not integrated by quadratures. Therefore, construction of the operator in the presence of a flow in explicit form is impossible. In this case the iteration process can be constructed in the following manner. In the actual data on the qualitative character of the velocity profile in a boundary layer we give the function $u^{(0)}(s, \eta)$, which satisfies the conditions in (1.18). We then solve the ordinary differential Equation (1.15) with the initial condition $\delta(0) = 0$. Then from (1.13) we find the profile $u^{(1)}(s, \eta)$ in first approximation, and then the process is repeated.

Let us give the form for writing the above algorithm that is final and more convenient for analytical and numerical iterations both in the variables s and η ,

$$\delta^{(n)}(s) \delta^{(n)'}(s) + \left[\beta(s) \frac{B^{(n)}(s, \infty)}{A^{(n)}(s, \infty)} + \frac{C^{(n)}(s, \infty)}{A^{(n)}(s, \infty)} \right] \delta^{(n)2}(s) + \frac{D(s, \infty)}{A(s, \infty)} \psi_w'(s) \delta^{(n)}(s) = \frac{1}{A^{(n)}(s, \infty)}$$

$$\delta(0) = 0 \quad (1.31)$$

$$u^{(n+1)}(s, \eta) = \frac{A^{(n)}(s, \eta)}{A^{(n)}(s, \infty)} + \left\{ \beta(s) \left[B^{(n)}(s, \eta) - \frac{A^{(n)}(s, \eta)}{A^{(n)}(s, \infty)} B^{(n)}(s, \infty) \right] + C^{(n)}(s, \eta) - \frac{A^{(n)}(s, \eta)}{A^{(n)}(s, \infty)} C^{(n)}(s, \infty) \right\} \delta^{(n)2}(s) + \left[D^{(n)}(s, \eta) - \frac{A^{(n)}(s, \eta)}{A^{(n)}(s, \infty)} D^{(n)}(s, \infty) \right] \psi_w'(s) \delta^{(n)}(s)$$

and in the variables x and η ,

$$\begin{aligned}
& \delta^{(n)} \delta^{(n)'}(x) + \left[\beta(x) \frac{B^{(n)}(x, \infty)}{A^{(n)}(x, \infty)} + \frac{C^{(n)}(x, \infty)}{A^{(n)}(x, \infty)} \right] \delta^{(n)2}(x) + \frac{D^{(n)}(x, \infty)}{A^{(n)}(x, \infty)} \psi_w'(x) \delta^{(n)}(x) = \frac{\nu U_e z^{2K}}{A^{(n)}(x, \infty)}, \delta(0) = 0 \\
& u^{(n)}(x, \eta) \nu U_e z^{2K} = \frac{A^{(n)}(x, \eta)}{A^{(n)}(x, \infty)} \nu U_e z^{2K} + \left\{ \beta(x) \left[B^{(n)}(x, \eta) - \frac{A^{(n)}(x, \eta)}{A^{(n)}(x, \infty)} B^{(n)}(x, \infty) \right] + \right. \\
& \quad \left. + C^{(n)}(x, \eta) - \frac{A^{(n)}(x, \eta)}{A^{(n)}(x, \infty)} C^{(n)}(x, \infty) \right\} \delta^{(n)2}(x) + \left[D^{(n)}(x, \eta) - \frac{A^{(n)}(x, \eta)}{A^{(n)}(x, \infty)} D^{(n)}(x, \infty) \right] \psi_w' \delta^{(n)}(x)
\end{aligned} \quad (1.32)$$

After finding the necessary number of iterations we can compute the friction on the wall from one of the formulas

$$\begin{aligned}
\tau^{(n)}(s) = \mu U_e^2 z^K \left\{ \frac{\tilde{\delta}(u)}{\delta^{(n)} A^{(n)}(s, \infty)} + \left[\beta(s) (\tilde{\delta}'(s) - \tilde{\delta}'(u)) \frac{B^{(n)}(s, \infty)}{A^{(n)}(s, \infty)} + \frac{\partial}{\partial s} \tilde{\delta}(u) - \tilde{\delta}(u) \frac{C^{(n)}(s, \infty)}{A^{(n)}(s, \infty)} \right] \delta^{(n)}(s) + \right. \\
\left. + \left[1 - \frac{D^{(n)}(s, \infty)}{A^{(n)}(s, \infty)} \tilde{\delta}(u) \right] \psi_w'(s) \right\}
\end{aligned} \quad (1.33)$$

$$\begin{aligned}
\tau^{(n)}(x) = \frac{\rho U_e}{2K} \left\{ \frac{\tilde{\delta}(u) \nu U_e z^{2K}}{A^{(n)}(x, \infty) \delta^{(n)}(x)} + \left[\beta(x) (\tilde{\delta}'(x) - \tilde{\delta}'(u)) \frac{B^{(n)}(x, \infty)}{A^{(n)}(x, \infty)} + \right. \right. \\
\left. \left. + \frac{\partial}{\partial x} \tilde{\delta}(u) - \tilde{\delta}(u) \frac{C^{(n)}(x, \infty)}{A^{(n)}(x, \infty)} \right] \delta^{(n)}(x) + \left[1 - \frac{D^{(n)}(x, \infty)}{A^{(n)}(x, \infty)} \tilde{\delta}(u) \right] \psi_w'(x) \right\} \quad (1.34) \quad \underline{18}
\end{aligned}$$

Thus, in the presence of a flow it is necessary in each cross section $s = \text{const}$ ($x = \text{const}$) to have, generally speaking, a numerical solution to the auxiliary ordinary differential equation for the function $\delta(s)$. However, in the numerical realization of this process, the appearance of an ordinary differential equation for the auxiliary function $\delta(s)$ does not complicate the computations very much.

COMMENTS

1. In any convergent method of successive approximations the choice of the initial approximation will determine the necessary number of iterations to obtain a specified accuracy. As the zero approximation we recommend using

the function $u^{(0)} = u^0(\eta)$, which depends only on η . Such a simple assignment of the zero approximation is connected with that property of the boundary layer equations which, with a sufficiently smooth function $Ue(x)$, the solution to Equation (1.10) will differ little from the quasi-self-similar solution obtained from Equation (1.10) if we delete the derivatives of s in it. With such a choice $u^{(0)}$ the dependence on s appears in the first approximation through the function $\delta(s)$. In Reference [1] two different initial functions were chosen: $u^0 = 1 - \exp(-\eta)$ and $u^{(0)} = \phi(\eta)$. The second function brought the iteration process to solution more rapidly.

2. It is important to mention that in this method of successive approximations we expect convergence of the variables x , y or x , y [see (1.23)], but not in the variables s , η or s , η , i.e.,

$$\lim_{n \rightarrow \infty} u^{(n)}(s, \frac{Ue x^k}{\delta^{(n)}(s)} y) = \tilde{u}(s, y)$$

The limit $\lim_{n \rightarrow \infty} u^{(n)}(s, \eta)$ may also not exist. The convergence of the method was proven experimentally by direct computation of a large number of iterations [1].

§ 2. BASIC EQUATIONS AND BOUNDARY CONDITIONS FOR PROBLEMS IN LAMINAR MULTI-COMPONENT BOUNDARY LAYER THEORY WITH HOMOGENEOUS AND HETEROGENEOUS REACTIONS FOR FLOW AROUND BODIES WITH IMPERMEABLE AND THERMOCHEMICALLY DISINTEGRATING WALLS

2.1 Basic Concepts

In flow of a gas around a body involving a high enthalpy of deceleration (for example, $h_0 \geq 1500 - 2000$ cal/gram) an intense heating of this gas takes place as a result of which its surface begins to become thermochemically disintegrated. Depending on the physico-chemical properties of the body's material and the conditions of the streamlining, the disintegration may take

place due to melting, evaporation, sublimation, heating, pyrolysis, mechanical disintegration, and most often due to the simultaneous occurrence of several of these processes. Ignoring mechanical transport, we shall assume that a mixture of gases flows to the boundary layer from the surface of the body, solid or liquid, due to the disintegration; this mixture may enter into chemical reaction with the gas of the boundary layer which itself in turn is a rather complex mixture of products of disintegration and ionization of the approaching current. Thus, we are concerned with the laminar boundary layer in which several homogeneous chemical reactions may take place simultaneously. In order for these reactions to be independent, it is necessary and sufficient that at least one of the components in each reaction not appear in any other reaction. Let N be the total number of components in the flow. Let the number of independent components, for which we can in particular take the chemical elements and the electron components, be N_e . Then all other components A_i ($i = 1, \dots, N_r = N - N_e$) can be expressed due to the N_r reactions through the base components A_j ($j = 1, \dots, N_e$), in particular in the following form

$$A_i = \sum_{j=N_e+1}^N v_{ij} A_j \quad (i = 1, \dots, N_r) \quad (2.1)$$

Here A_i and A_j are the chemical symbols of the components, v_{ij} are the stoichiometric coefficients, i.e., if we take the reaction products A_i ($i = 1, \dots, N_r$) which form as a result of the reactions from the base components A_j ($j = 1, \dots, N_e$) only once in each reaction and expand them from the left, and if we expand all other components from the right and divide by the stoichiometric coefficient in front of the reaction product, the choice of independent reactions can be written in the form of (2.1). In accordance with the first law of thermodynamics, the heats of the reactions which are necessary for formation of the mass unit of the products A_i with constants p and T , are determined by equations

$$\bar{h}_i = \sum_{j=N_k+1}^N \nu_{ij} \frac{m_i}{m_j} \bar{h}_j - Q_i(T) \quad (i=1, \dots, N_k) \quad (2.2)$$

where $Q_i(T)$, $[Q_i]$ = cal/gram is the heat of the i -th reaction per gram of the product A_i , m_i is the molecular weight of the i -th component, \bar{h}_i , $[h_i]$ = cal/gram is the specific enthalpy of the i -th component, determined by the differential equation

$$\frac{d\bar{h}_i}{dT} = C_{pi}(T) \quad (i=1, \dots, N)$$

where C_{pi} , $[C_{pi}]$ = cal/gram·deg is the specific enthalpy of the i -th component.

If the heats of the reactions are given, then from (2.2) it follows that N_r of the enthalpies \bar{h}_i are expressed linearly through the enthalpy of the base components \bar{h}_j .

We further assume that the mixture of gases in the boundary layer is a mixture of ideal gases, i.e., the enthalpy of the mixture is equal to

$$\bar{h} = \sum_{k=1}^N C_k \bar{h}_k, \quad C_i = \frac{\rho_i}{\rho} \quad (i=1, \dots, N), \quad \rho = \sum_{k=1}^N \rho_k \quad (2.3)$$

where ρ_i , $[\rho_i]$ = gram/cm³ is the mass density of the i -th component, C_i is the mass concentration of the i -th component, ρ , $[\rho]$ = gram/cm³ is the density of the mixture.

Let us compute the differential of the enthalpy of the mixture by using (2.2)

$$\begin{aligned} d\bar{h} &= C_p dT + \sum_{k=1}^N \bar{h}_k dC_k = C_p dT + \sum_{k=1}^{N_k} \bar{h}_k dC_k + \sum_{k=N_k+1}^N \bar{h}_k dC_k = \\ &= C_p dT + \sum_{j=N_k+1}^N \bar{h}_j dC_j^* - \sum_{k=1}^{N_k} Q_k dC_k = d \left(\sum_{j=N_k+1}^N C_j^* \bar{h}_j - \sum_{k=1}^{N_k} C_k Q_k \right) \end{aligned} \quad (2.4)$$

$$C_p = \sum_{k=1}^N C_{pk} C_k$$

where C_{pj} , $[C_{pj}] = \text{cal/gram} \cdot \text{deg}$ is the specific enthalpy of the j -th component, c_j^* is the concentration of the j -th element, equal by definition to

$$C_j^* = C_j + \sum_{k=1}^{N_e} \nu_{kj} \frac{m_j}{m_k} C_k \equiv \sum_{k=1}^N m_{jk} C_k, \quad m_{jk} = \frac{\nu_{kj} m_j}{m_k}, \quad \sum_{j=N_e+1}^N C_j^* \equiv 1 \quad \begin{matrix} (j=1, \dots, N_e) \\ (i=1, \dots, N_e) \end{matrix} \quad (2.5)$$

Here m_{ji} is the amount of the mass of the j -th element in the i -th component. By definition the concentration of C_j^* is the concentration of the j -th element, regardless of what component it is located in. For example, if only chemical reactions (no ionization) occur in the flow and as the base components we select the chemical elements, then C_j^* represents the concentration of the chemical element regardless of what component it is located in. In the case of an arbitrary choice of the base components (A_j are not necessarily chemical elements) the concentrations C_j^* ($j = 1, \dots, N_e$) will be termed concentrations of base elements or simply concentrations of elements. In the presence of ionization the electron component will always be included in the group of base elements.

As an example, let us look at a mixture composed of elements of O, N, E (electron), in which, for example, six independent reactions take place



The concentration of elements here will be given by the formulas

$$\begin{aligned} C_o^* &= C_o + \frac{2m_o}{m_{\text{O}_2}} C_{\text{O}_2} + \frac{m_o}{m_{\text{NO}}} C_{\text{NO}} + \frac{m_o}{m_{\text{O}^+}} C_{\text{O}^+} + \frac{m_o}{m_{\text{NO}^+}} C_{\text{NO}^+} \\ C_N^* &= C_N + \frac{2m_N}{m_{\text{N}_2}} C_{\text{N}_2} + \frac{m_N}{m_{\text{NO}}} C_{\text{NO}} + \frac{m_N}{m_{\text{N}^+}} C_{\text{N}^+} + \frac{m_N}{m_{\text{NO}^+}} C_{\text{NO}^+} \\ C_E^* &= C_E - \frac{m_E}{m_{\text{O}^+}} C_{\text{O}^+} - \frac{m_E}{m_{\text{N}^+}} C_{\text{N}^+} - \frac{m_E}{m_{\text{NO}^+}} C_{\text{NO}^+} \end{aligned} \quad (2.7)$$

Let us further look at the flow under the condition that the mixture of gases /10 at each point is quasi-neutral, i.e., if n_i^* is the number of particles of the

i-th sort per unit volume, e_i is the charge of the i-th particle, then the condition of quasi-neutrality can be written as

$$\sum_{i=1}^N n_i^* e_i = 0 \quad \text{or} \quad \sum_{k=1}^N x_k e_k \quad \text{or} \quad \sum_{k=1}^N \frac{C_k}{m_k} e_k = 0 \quad (2.8)$$

$$x_i = \frac{n_i}{n} = \frac{m_i}{m} C_i \quad (i = 1, \dots, N)$$

$$n = \sum_{k=1}^N n_k, \quad n_i^* = N_A \cdot n, \quad \frac{1}{m} = \sum_{k=1}^N \frac{C_k}{m_k} = \sum_{k=1}^N x_k m_k$$

(2.8)

where x_i is the molar (numerical) concentration of the i-th component, N_A is the Avagadro number.

The condition of quasi-neutrality (2.8) may be written in the form

$$C_E + \sum_{k=1}^{N_I} \frac{m_E}{m_{Ik}} \frac{e_k}{e} C_{Ik} = 0 \quad (2.10)$$

where N_I is the number of ions in the mixture, e is the electron charge. But the left-hand side of Condition (2.10) is simply the concentration of the electron elements. Therefore, in the framework of the assumption of quasi-neutrality, the concentration of the E element is always equal to zero

$$C_E^* = C_E + \sum_{k=1}^{N_I} \frac{e_k}{e} \frac{m_E}{m_{Ik}} C_{Ik} = 0 \quad (2.11)$$

Equation (2.11) may be assumed to be an integral of the basic equations of motion, which will be given below.

If the concentrations of elements are retained in the current as a result of the conditions of the problem, i.e., $C_j^* = \text{const}$ ($j = 1, \dots, N_e$), then from (2.4) it follows that

$$dh = c_p dT - \sum_{k=1}^{N_e} Q_k(T) dC_k = \sum_{j=N_e+1}^N c_j^* dh_j - \sum_{k=1}^{N_e} d(C_k Q_k) \quad (2.12)$$

This will be the case, for example, in the case of gas flow in the absence of diffusion. However, in real cases with diffusion, even without the addition of supplementary components to the flow from the side of the boundaries, the concentrations C_j^* will vary from point to point, i.e., the liquid particle in a real flow from the viewpoint of thermodynamics will always be an open system, and the change in enthalpy dh for it will be computed from Formula (2.4) rather than from (2.12).

2.2 Basic System of Equations for a Laminar Multi-Component Boundary Layer

The equations of a laminar asymptotically thin two-dimensional stationary boundary layer on two-dimensional paths (plane-parallel motion) and the bodies of revolution (axisymmetric motion), allowing for the chemical reactions and the ionization reactions without allowing for the external electromagnetic fields and the radiation field, have the form [6]:

Equation of continuity:

$$\frac{\partial}{\partial x}(\rho u r^k) + \frac{\partial}{\partial y}(\rho v r^k) = 0 \quad (2.13)$$

Momentum equation projected onto the y-axis:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho \mu \frac{d\mu}{dx} + \frac{\partial}{\partial y}(\mu \frac{\partial u}{\partial y}) \quad (2.14)$$

Equation of diffusion of the components:

$$\rho u \frac{\partial c_i}{\partial x} + \rho v \frac{\partial c_i}{\partial y} + \frac{\partial J_i}{\partial y} = \dot{W}_i \quad (i=1, \dots, N) \quad (2.15)$$

along with the Stefan - Maxwell relationships

$$\frac{1}{x_i} \frac{\partial x_i}{\partial y} = \sum_{j=1}^N x_j a_{ij} \left(\frac{J_j}{\rho_j} - \frac{J_i}{\rho_i} \right) + \frac{e_i F}{kT} + \frac{\partial \ln T}{\partial y} \sum_i^r \quad (2.16) \quad \underline{11}$$

or

$$\begin{aligned} \frac{1}{\alpha} \frac{\partial \alpha}{\partial y} = & - \frac{\gamma_i}{\beta_i} \sum_{j=1}^N x_j a_{ij} + \sum_{j=1}^N \left[x_j a_{ij} + \sum_{k=1}^N (g_j x_k - c_k x_j) \right] \frac{\gamma_j}{\beta_j} + \\ & + \left(e_i - \sum_{k=1}^N c_k e_k \right) \frac{E}{RT} + \left(\sum_{i=1}^N \gamma_i - \sum_{k=1}^N c_k \sum_{i=1}^N \gamma_i \right) \frac{\partial \ln T}{\partial y} \\ \alpha_i = & \frac{m_i}{m} \alpha, \quad \gamma_i = \beta_i (v_i - v) \equiv \beta_i V_i, \quad \sum_{i=1}^N \gamma_i = \sum_{j=1}^N x_j a_{ij} \left(\frac{\mathcal{D}_j^T}{\beta_j} - \frac{\mathcal{D}_i^T}{\beta_i} \right) \quad (2.17) \\ & \sum_{i=1}^N \mathcal{D}_i^T = 0, \quad a_{ij} = a_{ji} \end{aligned}$$

The heat flux equation:

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = - \frac{\partial q}{\partial y} - \sum_{k=1}^N \gamma_k \frac{\partial h_k}{\partial y} - \beta_e u_e \frac{du}{dx} u + \mu \left(\frac{\partial u}{\partial y} \right)^2 - \sum_{k=2}^N h_k \dot{w}_k \quad (2.18)$$

along with the expression for the reduced heat flux⁽¹⁾

$$\begin{aligned} q = & - \lambda \frac{\partial T}{\partial y} + R_A T \sum_{j=1}^N \sum_{k=1}^N \frac{x_j \mathcal{D}_k^T}{m_k} a_{kj} (V_k - V_j) \\ \lambda = & \lambda' - R_A \sum_{k=1}^N \sum_{j=1}^N \frac{x_j \mathcal{D}_k^T}{m_k} a_{kj} \left(\frac{\mathcal{D}_k^T}{\beta_k} - \frac{\mathcal{D}_j^T}{\beta_j} \right) = \\ = & \lambda' - \frac{1}{2} \cdot R_A \cdot n \sum_{k=1}^N \sum_{j=1}^N x_k x_j a_{kj} \left(\frac{\mathcal{D}_k^T}{\beta_k} - \frac{\mathcal{D}_j^T}{\beta_j} \right)^2 \end{aligned} \quad (2.19)$$

and the expression for the total heat flux along the y-axis

$$\mathcal{H}_q = q + \sum_{k=2}^N h_k \gamma_k \quad (2.21)$$

The equation of state

$$\rho = \rho \frac{RAT}{m}, \quad \frac{1}{m} = \sum_{k=1}^N \frac{c_k}{m_k} = \sum_{k=1}^N x_k m_k \quad (2.22)$$

(1) Terminology of I. Progozhin [7].

Here x is the linear coordinate along the surface of the body, y is that along the normal to the body's surface, i.e., we have chosen the usually accepted coordinate system here: the normals to the generatrices of the body and their orthogonal trajectories, u , v are the projections of the velocity vector onto the x and y -axes, $r(x)$ is the radius of the cross section of the body of revolution, $k = 0$ with flow around a two-dimensional path (plane-parallel problem), $k = 1$ for flow around a body of revolution (axisymmetrical problem), J_i [J_i] = gram/cm²sec is the mass diffusion flow of the i -th component in the direction of the y -axis, W_i , [W_i] = gram/cm³sec is the mass rate of formation of the i -th component per unit volume per unit of time due to all the possible reactions in the boundary layer, V_i is the diffusion velocity of the i -th component, a_{ij} , [a_{ij}] = sec/cm² is the coefficient of resistance, the expressions for which will be given below, k is the Boltzmann constant, e_i is the electric charge of the i -th component, E is the electric field strength which arises due to the separation of charges (the external field is equal to zero), D_i^T , [D_i^T] = gram/cm·sec is the coefficient of thermal diffusion of the i -th component, T is the temperature, $U_e(x)$ is the specified velocity of the nonviscous flow on a given body, P is the pressure, m is the mean molecular weight, n is the number of moles per unit volume, μ is the coefficient of dynamic viscosity, λ is the ordinary coefficient of thermal conductivity, the coefficient λ' is computed in the kinetic theory of gases through the expansion coefficients in the Sonin polynomial [8], R_A is the absolute gas constant, N is the number of components in the mixture; the other symbols are given in Section 2.1. The last term in (2.16) and (2.17) expresses the effect of thermal diffusion, the second term in (2.19) is the appearance of the diffusion thermal effect. Both these effects, especially the latter, have /12 little influence on the solution to the problem in the case of a mixture of neutral gases, since the coefficients of thermal diffusion D_i' ($i = 1, \dots, N$) in this case are small. In the presence of significant ionization there is a suspicion that these effects may make a considerable contribution to the solution of the problem. However this question at the present time is still not clear. The effect of barodiffusion, just as the effect of the viscous transport of momentum, will occur on the strength of the approxima-

tions in the boundary layer theory⁽²⁾. The coefficients of resistance a_{ij} (K , $j = 1, 2, \dots, N$) have a different expression depending on which law the interaction of the component particles obeys during their convergence. With a Coulomb interaction of the particles, we have in first approximation the computations of the transport coefficients [8]

$$\frac{1}{a_{ij}} = [D_{ij}]_1 = \frac{3}{16} \left[\frac{2kT(m_i + m_j)}{\pi m_i m_j} \right]^{1/2} \frac{kT}{P} \left(\frac{2kT}{e_i e_j} \right)^2 \frac{1}{2 \ln \Lambda_{ij}}$$

$$[D_{ij}] = \frac{\text{cm}^2}{\text{sec}}, \quad [m_i] = \text{gram}, \quad [T] = ^\circ K, \quad [P] = \frac{\text{g}}{\text{cm}^3 \text{sec}^2}$$

where Λ_{ij} is the Coloumb logarithm.

With the interaction between the particles and the Lennard - Jones Potential (6 - 12) [8].

$$\frac{1}{a_{ij}} = [D_{ij}]_1 = \frac{3}{16} \left[\frac{2kT(m_i + m_j)}{\pi m_i m_j} \right]^{1/2} \frac{kT}{P} \frac{1}{\sigma_{ij}^2 \Omega_{ij}^{(2,2)*}(\epsilon_{ij})}$$

$$\sigma_{ij} = \frac{1}{2}(\sigma_i + \sigma_j), \quad \epsilon_{ij} = \sqrt{\epsilon_i \epsilon_j}, \quad \Omega_{ij} = \frac{kT}{\epsilon_{ij}}$$

$$[D_{ij}] = \frac{\text{cm}^2}{\text{sec}}, \quad [m_i] = \text{gram}, \quad [T] = ^\circ K, \quad [P] = \frac{\text{g}}{\text{cm}^3 \text{sec}^2} \quad [\sigma] = \text{cm}$$

where σ_i is the distance between particles of i -th kind for which the energy of the interaction is equal to zero, ϵ_i is the absolute value of the maximal energy of attraction, $\Omega_{ij}^{(2,2)*}(\epsilon_{ij})$ is a known function which depends weakly on the temperature when $[\epsilon_{ij}] \gg 3$ [8].

With the interaction between rigid spheres having diameters σ_i and σ_j we have

$$\frac{1}{a_{ij}} = [D_{ij}]_1 = \frac{3}{16} \left[\frac{2kT(m_i + m_j)}{\pi m_i m_j} \right]^{1/2} \frac{kT}{P} \frac{1}{\sigma_{ij}^2}, \quad \sigma_{ij} = (\sigma_i + \sigma_j)$$

(2) Proof of the small influence of the viscous momentum transport in approximation of the boundary layer theory was given in the thesis of G. A. Danilin, a student at Moscow State University.

It should be mentioned that, unlike the last formula for σ_{ij} , the expression for σ_{ij} in the case of interaction with the Lennard - Jones potential, $\sigma_{ij} = 1/2 (\sigma_i + \sigma_j)$, is a semiempirical one, just as the formula for ϵ_{ij} . The System (2.13) - (2.22) along with (2.11) represents a nonlinear system of $2N + 5$. Equations (2.13) - (2.18), (2.22) for $2N + 5$ unknown functions: $u, v, T, \rho, C_i (i=1, 2, \dots, N), \gamma_i (i=1, 2, \dots, N)$. The pressure P is given as a function of x .

In an actual solution to the problem, it is often convenient to introduce the concentrations of elements (2.5). Let us obtain equations for them.

In the presence of arbitrary reactions in the flow, the element as such does not disappear and does not appear in the flow. Therefore if the equations of diffusion of the components (2.15) are multiplied by the mass of the j -th element in the i -th component, i.e., by $m_{ji} = v_{ij}m_j/m_i$ and are summed over all components, we then find N_e equations of diffusion of the elements in the form,

$$\begin{aligned} \rho \left(u \frac{\partial c_j^*}{\partial x} + v \frac{\partial c_j^*}{\partial y} \right) + \frac{\partial \gamma_j^*}{\partial y} &= 0 \\ \gamma_j^* &= \gamma_j + \sum_{k=1}^{N_e} \frac{v_{kj} m_j}{m_k} \gamma_k \quad (j=1, \dots, N_e = N - N_z) \end{aligned} \quad (2.23)$$

since on the strength of the above

$$\sum_{k=1}^N m_{jk} W_k \equiv 0 \quad (j=1, \dots, N_e) \quad (2.24)$$

Let us mention that the equations in (2.23) are valid always, whether reactions take place or not and whether they take place at a finite rate or at an infinite rate (chemical balance).

If we write Equation (2.23) for the electron component, then on the strength of (2.11) we find

$$\frac{\partial \mathcal{J}_E^*}{\partial y} = 0 \quad \mathcal{J}_E^* = f(x)$$

where $f(x)$ is an arbitrary function. For its determination we compute the total charge transport across the boundary layer

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$$\begin{aligned} \sum_{k=1}^N n_k^* e_k \bar{v}_k &= N_A \sum_{k=1}^N n_k e_k \bar{V}_k = N_A \sum_{k=1}^N \frac{e_k}{m_k} \mathcal{J}_k = N_A \left(\sum_{k=1}^{N_I} \frac{e_{Ik}}{m_{Ik}} \mathcal{J}_{Ik} - \frac{e}{m_E} \mathcal{J}_E \right) = \\ &= N_A \frac{e}{m_E} \left(\sum_{k=1}^{N_I} \frac{e_{Ik}}{e} \frac{m_E}{m_{Ik}} \mathcal{J}_{Ik} - \mathcal{J}_E \right) = -N_A \frac{e}{m_E} \mathcal{J}_E^* \end{aligned} \quad (2.25)$$

where v_i is the mean statistical velocity of the i -th component. Hence, it follows that if the ionized gas does not contact the external conductors, in the future we shall assume that $\mathcal{J}_E^* = 0$ at the boundary, whence $f(x) \equiv 0$. Consequently, we shall have still one more integral

$$\mathcal{J}_E^* = 0 \quad (2.26)$$

Furthermore, we always have

$$\sum_{k=1}^N c_k \equiv 1, \quad \sum_{k=1}^N \mathcal{J}_k = 0 \quad (2.27)$$

which follows from the determination of the concentrations and flows. Thus, in our formulation, four integrals exist (2.11), (2.26) and (2.27) which we shall use in the future. The equations in (2.23) are preferable to those in (2.15), since these latter contain a non-zero right-hand side (mass sources). However, the equations in (2.23) are smaller than N . The missing $N - N_e = N_r$ equations for the concentrations will be, in the case of reactions taking place at finite rates, the equations of diffusion for the reaction products

$$\rho \left(u \frac{\partial c_i}{\partial x} + v \frac{\partial c_i}{\partial y} \right) + \frac{\partial \mathcal{J}_i}{\partial y} = \dot{W}_i \quad (i = 1, \dots, N_r) \quad (2.28)$$

In the case of reactions taking place with infinite rapidity in the flow (chemical balance), these will be the conditions of equilibrium

$$\prod_{k=1}^N P_k^{\nu_k} = K_{p,i}(T) \quad (i = 1, \dots, N_r) \quad (2.29)$$

where P_i is the partial pressure of the i -th component, $K_{p,i}(T)$ is a known constant of equilibrium of the i -th reaction. In accordance with the description of the reaction in the form of (2.1), these conditions are rewritten as

$$\prod_{j=N_r+1}^N \frac{P_j^{\nu_j}}{P_i} = K_{p,i}(T) \quad (i = 1, \dots, N_r) \quad (2.30)$$

By converting in the last expression to mass concentrations, we ultimately find

$$\prod_{j=N_r+1}^N \left(\frac{c_j}{m_j} \right)^{\nu_j} \left(\frac{c_i}{m_i} \right)^{-1} = \frac{K_{p,i}(T)}{P^{\nu_{m,i}}}, \quad \nu_i = \sum_{j=N_r+1}^N \nu_{ij} - 1 \quad (i = 1, \dots, N_r) \quad (2.30')$$

In the case of chemical balance from Equations (2.15), after solving the problem, we can compute the left-hand side and in the same way compute the distribution of mass sources W_i ($i = 1, \dots, N$) due to the balanced chemical reactions. However, as a rule, this is not done in the literature. To conclude this section, let us transform the heat flux equation (2.18) to a more convenient form; let us introduce into it explicitly the heats of the reactions. This last term in Equation (2.18), allowing for (2.2) and the equations in (2.15), may be represented as

$$\begin{aligned}
 -\sum_{k=1}^N h_k W_k &= -\sum_{k=1}^{N_L} h_k W_k - \sum_{k=N_L+1}^N h_k W_k = \sum_{k=1}^{N_L} Q_k W_k = \\
 &= \sum_{k=1}^{N_L} Q_k(T) \left(\rho u \frac{\partial C_k}{\partial x} + \rho v \frac{\partial C_k}{\partial y} \right) + \sum_{k=1}^{N_L} Q_k(T) \frac{\partial Y_k}{\partial y}
 \end{aligned} \quad (2.31)$$

since on the strength of conservation of the mass of elements in the Reactions (2.1) the following conditions are always satisfied:

$$\dot{W}_j + \sum_{k=1}^{N_L} \nu_{kj} \frac{m_j}{m_k} \dot{W}_k \equiv 0 \quad (j = 1, \dots, N_e) \quad (2.32)$$

If we substitute Expression (2.31) into (2.18), we find

$$\begin{aligned}
 &\rho u \left[c_p \frac{\partial T}{\partial x} - \sum_{k=1}^{N_L} Q_k \frac{\partial C_k}{\partial x} \right] + \rho v \left[c_p \frac{\partial T}{\partial y} - \sum_{k=1}^{N_L} Q_k \frac{\partial C_k}{\partial y} \right] = \\
 &= \frac{\partial}{\partial y} \left(-q + \sum_{k=1}^{N_L} Q_k Y_k \right) - \rho_e U_e \frac{dU_e}{dx} + \mu \left(\frac{\partial u}{\partial y} \right)^2 - \sum_{k=1}^N Y_k \frac{\partial h_k}{\partial y} - \sum_{k=1}^{N_L} Y_k \frac{\partial Q_k}{\partial y} \quad 114
 \end{aligned}$$

Using (2.2) the last two terms are transformed into simpler form

$$\begin{aligned}
 \sum_{k=1}^N Y_k \frac{\partial h_k}{\partial y} + \sum_{k=1}^{N_L} Y_k \frac{\partial Q_k}{\partial y} &= \sum_{k=1}^N Y_k \sum_{j=N_L+1}^N \nu_{kj} \frac{m_j}{m_k} \frac{\partial h_j}{\partial y} + \sum_{k=N_L+1}^N Y_k \frac{\partial h_k}{\partial y} = \\
 &= \sum_{j=N_L+1}^N Y_j^* \frac{\partial h_j}{\partial y} = \frac{\partial T}{\partial y} \sum_{j=N_L+1}^N c_{p_j} Y_j^*
 \end{aligned}$$

after which the heat flux equation will be

$$\begin{aligned}
 &\rho u \left[c_p \frac{\partial T}{\partial x} - \sum_{k=1}^{N_L} Q_k \frac{\partial C_k}{\partial x} \right] + \rho v \left[c_p \frac{\partial T}{\partial y} - \sum_{k=1}^{N_L} Q_k \frac{\partial C_k}{\partial y} \right] = \\
 &= \frac{\partial}{\partial y} \left(-q + \sum_{k=1}^{N_L} Q_k Y_k \right) - \rho_e U_e \frac{dU_e}{dx} + \mu \left(\frac{\partial u}{\partial y} \right)^2 - \frac{\partial T}{\partial y} \sum_{j=N_L+1}^N c_{p_j} Y_j^* \quad (2.33)
 \end{aligned}$$

In such a form of the heat flux equations, it is easy to compute the influence of the heats of the reactions on the temperature.

If Equation (2.14) is multiplied by u and added to the heat flux Equation (2.33), we then find the equation of energy described through the temperature

$$\begin{aligned} & \rho u \left[c_p \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left(\frac{u^2}{2} \right) - \sum_{k=1}^{N_k} Q_k \frac{\partial c_k}{\partial x} \right] + \rho v \left[c_p \frac{\partial T}{\partial y} + \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) - \sum_{k=1}^{N_k} Q_k \frac{\partial c_k}{\partial y} \right] = \\ & = \frac{\partial}{\partial y} \left[-q + \sum_{k=1}^{N_k} Q_k T_k + \mu \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) \right] - \frac{\partial T}{\partial y} \sum_{j=N_k+1}^N c_{pj} T_j^* \end{aligned} \quad (2.34)$$

The equation of energy in such form can be conveniently used in the presence of chemical reactions occurring at finite or infinite rates. For "frozen" flows the heat flux equation is obtained from (2.18) if we set $W_i = 0$ ($i = 1, \dots, N$) in it; the equation of energy for a "frozen" flow will be

$$\rho u \left[c_p \frac{\partial T}{\partial x} + \frac{\partial}{\partial x} \left(\frac{u^2}{2} \right) \right] + \rho v \left[c_p \frac{\partial T}{\partial y} + \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) \right] = \frac{\partial}{\partial y} \left[-q + \mu \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) \right] - \frac{\partial T}{\partial y} \sum_{k=1}^N c_{pk} T_k \quad (2.35)$$

Let us introduce into (2.34) the concept of total enthalpy in the presence of chemical reactions. With the flow of an ideal gas around a body, without allowing for radiation along the jet stream on the surface, there remains a constant value (total enthalpy of the advancing current)

$$H_e = h_e + \frac{U_e^2}{2} \equiv \sum_{k=1}^N c_{ke} h_{ke} + \frac{U_e^2}{2} = h_\infty + \frac{U_\infty^2}{2} \quad (2.36)$$

which with the aid of (2.2) can be represented in the form

$$H_e = \sum_{j=N_k+1}^N c_{je}^* h_{je} - \sum_{k=1}^{N_k} c_{ke} Q_k (T_e) + \frac{U_e^2}{2} \quad (2.37)$$

where the index "e" refers to the parameters of the nonviscous gas on the

wall or in the boundary layer scale to the conditions on the outermost boundary of the boundary layer, the index " ∞ " refers to the conditions in the advancing flow. It is convenient to have such a value also for the flow in a boundary layer in the presence of chemical reactions. Using (2.2), the convective part in (2.34) can be represented as, for example, /15

$$\begin{aligned} \rho \frac{\partial I}{\partial x} - \sum_{k=1}^{N_k} Q_k \frac{\partial C_k}{\partial x} &= \sum_{k=1}^N c_k \frac{\partial h_k}{\partial x} - \sum_{k=1}^{N_k} \frac{\partial}{\partial x} (c_k Q_k) + \sum_{k=1}^{N_k} c_k \frac{\partial Q_k}{\partial x} = \\ &= \sum_{j=N_k+1}^N c_j^* \frac{\partial h_j}{\partial x} - \sum_{k=1}^{N_k} \frac{\partial}{\partial x} (c_k Q_k) = \sum_{j=N_k+1}^N \frac{\partial}{\partial x} c_j^* h_j - \sum_{j=N_k+1}^N h_j \frac{\partial c_j^*}{\partial x} - \sum_{k=1}^{N_k} \frac{\partial}{\partial x} (c_k Q_k) \\ &= \sum_{j=N_k+1}^N \frac{\partial}{\partial x} c_j^* h_j - \sum_{k=1}^{N_k} \frac{\partial}{\partial x} (c_k Q_k) - \sum_{j=N_k+1}^N (c_j^* - c_j) \frac{\partial h_j}{\partial x} \end{aligned}$$

Here we have used the conditions $\frac{dc_j^*}{dx} = 0$, which express the fact that along the jet stream of an external nonviscous flow the concentrations of elements are retained. Now if in Equation (2.34) we introduce a new unknown function (total enthalpy) instead of the temperature T ,

$$H = \sum_{j=N_k+1}^N c_j^* h_j - \sum_{k=1}^{N_k} c_k Q_k + \frac{u^2}{2} \quad (2.38)$$

which on the strength of (2.36) and (2.37) will be constant at the outermost boundary of the boundary layer

$$He = \text{const} \quad (2.39)$$

it then assumes the following form

$$\begin{aligned} \rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} &= \frac{\partial}{\partial y} \left[-q + \frac{\lambda}{c_p} \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) + \sum_{k=1}^{N_k} Q_k T_k + \frac{\lambda}{c_p} (G-1) \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) \right] + \\ &+ \sum_{j=N_k+1}^N (c_j^* - c_j) \left[\rho u \frac{\partial h_j}{\partial x} + \rho v \frac{\partial h_j}{\partial y} \right] - \sum_{j=N_k+1}^N h_j^* \frac{\partial c_j}{\partial y} \end{aligned} \quad (2.40)$$

or by using Equation (2.33), the following form

$$\begin{aligned} \rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} + \sum_{j=N_r+1}^N [\rho u \frac{\partial}{\partial x} (c_j^* - c_j^e) h_j + \rho v \frac{\partial}{\partial y} (c_j^* - c_j^e) h_j + \frac{\partial}{\partial y} \mathcal{J}_j^* h_j] = \\ = \frac{\partial}{\partial y} \left[-q + \frac{\lambda}{c_p} \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) + \sum_{k=1}^{N_k} Q_k \mathcal{J}_k + \frac{\lambda}{c_p} (\sigma - 1) \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) \right] \end{aligned} \quad (2.41)$$

where $\sigma = \frac{\mu c_p}{\lambda}$ is the Prandtl number.

Let us transform the right-hand side of Equation (2.41) such that the function H enters into it. We find by using (2.2)

$$\begin{aligned} \lambda \frac{\partial T}{\partial y} = \frac{\lambda}{c_p} \sum_{k=1}^{N_k} c_k \frac{\partial h_k}{\partial y} = \frac{\lambda}{c_p} \left(\sum_{j=N_r+1}^N c_j^* \frac{\partial h_j}{\partial y} - \sum_{k=1}^{N_k} c_k \frac{\partial Q_k}{\partial y} \right) = \\ = \frac{\lambda}{c_p} \left[\sum_{j=N_r+1}^N \frac{\partial}{\partial y} (c_j^e h_j) - \sum_{k=1}^{N_k} \frac{\partial}{\partial y} (c_k Q_k) \right] + \frac{\lambda}{c_p} \left[\sum_{j=N_r+1}^N (c_j^* - c_j^e) \frac{\partial h_j}{\partial y} + \sum_{k=1}^{N_k} Q_k \frac{\partial c_k}{\partial y} \right] \end{aligned}$$

Then Equation (2.41) can be written in the following form

$$\begin{aligned} \rho u \frac{\partial}{\partial x} (H - H_e) + \rho v \frac{\partial}{\partial y} (H - H_e) + \sum_{j=N_r+1}^N \left\{ \rho u \frac{\partial}{\partial x} [(c_j^* - c_j^e) h_j] + \right. \\ \left. + \rho v \frac{\partial}{\partial y} [(c_j^* - c_j^e) h_j] + \frac{\partial}{\partial y} (\mathcal{J}_j^* h_j) \right\} = \\ = \frac{\partial}{\partial y} \left\{ \frac{\lambda}{c_p} \left[\frac{\partial}{\partial y} (H - H_e) + (\sigma - 1) \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) - \sum_{j=N_r+1}^N (c_j^e - c_j^*) \frac{\partial h_j}{\partial y} \right] + \right. \\ \left. + \sum_{k=1}^{N_k} Q_k \left(\mathcal{J}_k + \frac{\lambda}{c_p} \frac{\partial c_k}{\partial y} \right) - R_A T \sum_{k=1}^N \sum_{j=1}^N \frac{x_j D_k^0 a_{kj}}{m_k} (V_k - V_j) \right\} \end{aligned} \quad (2.42)$$

Let us note that with identical specific heats of the elements, or when the element is alone, the sums of the left and the right for j disappear from $N_r + 1$ to N. In practice, for the majority of sets of elements, this assumption is satisfied quite well. /16

The double sum in (2.42) which allows for the diffusion thermal effect can be feasibly transformed to a more graphic form by excluding from it the diffusion currents of the base components \mathcal{J}_j ($j = N_r + 1, \dots, N$) using the definitions in (2.23); then it assumes the following form

$$\sum_{k=1}^N \sum_{j=1}^N \frac{x_j \mathcal{D}_k^T}{m_k} a_{kj} (V_k - V_j) = - \sum_{k=1}^{N_e} G_k^T \frac{\mathcal{I}_k}{m_k} - \sum_{j=N_e+1}^N \sum_j^T \frac{\mathcal{I}_j^*}{m_j}$$

where

$$G_i^T = \sum_i^T - \sum_{j=N_e+1}^N v_{ij} \sum_j^T$$

and Equations (2.41) and (2.42) will correspondingly be

$$\begin{aligned} & \rho u \frac{\partial}{\partial x} (H - H_e) + \rho v \frac{\partial}{\partial y} (H - H_e) + \sum_{j=N_e+1}^N \left\{ \rho u \frac{\partial}{\partial x} [(c_j^* - c_{je}^*) h_j] + \rho v \frac{\partial}{\partial y} [(c_j^* - c_{je}^*) h_j] + \right. \\ & \left. + \frac{\partial}{\partial y} (\mathcal{I}_j^* h_j) \right\} = \frac{\partial}{\partial y} \left[\lambda \frac{\partial T}{\partial y} + \frac{\lambda}{c_p} \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) + \frac{\lambda}{c_p} (G-1) \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) + \sum_{k=1}^{N_e} (Q_k \mathcal{I}_k + \frac{R_A T}{m_k} G_k^T \mathcal{I}_k) + \right. \\ & \left. + R_A T \sum_{j=N_e+1}^N \sum_j^T \frac{\mathcal{I}_j^*}{m_j} \right] \end{aligned} \quad (2.41)$$

$$\begin{aligned} & \rho u \frac{\partial}{\partial x} (H - H_e) + \rho v \frac{\partial}{\partial y} (H - H_e) + \sum_{j=N_e+1}^N \left\{ \rho u \frac{\partial}{\partial x} [(c_j^* - c_{je}^*) h_j] + \rho v \frac{\partial}{\partial y} [(c_j^* - c_{je}^*) h_j] + \right. \\ & \left. + \frac{\partial}{\partial y} (\mathcal{I}_j^* h_j) \right\} = \frac{\partial}{\partial y} \left\{ \frac{\lambda}{c_p} \left[\frac{\partial}{\partial y} (H - H_e) + (G-1) \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) - \sum_{j=N_e+1}^N (c_{je}^* - c_j^*) \frac{\partial h_j}{\partial y} \right] + \right. \\ & \left. + \sum_{k=1}^{N_e} \left[Q_k (\mathcal{I}_k + \frac{\lambda}{c_p} \frac{\partial c_k}{\partial y}) \right] + R_A T \sum_{j=N_e+1}^N \sum_j^T \frac{\mathcal{I}_j^*}{m_j} + \sum_{k=1}^{N_e} \frac{R_A T G_k^T}{m_k} \mathcal{I}_k \right\} \end{aligned} \quad (2.42')$$

Let us exclude the electric field E , by using the condition of plasma quasi-neutrality (2.8). For this let us differentiate Condition (2.8), described through the molar concentrations ⁽³⁾

$$\sum_{j=1}^N e_j \frac{\partial x_j}{\partial y} = 0$$

(3) We can look at Conditions (2.8) as integrals of the basic system of equations. Therefore, they can be differentiated on the strength of the solutions.

and substitute Expression (2.16) into it. Hence, then we find the electric field which arises due to separation of the charges

$$\frac{E}{kT} = - \left(\sum_{k=1}^N x_k e_k^2 \right)^{-1} \left[\sum_{j=1}^N \frac{\gamma_j x_j}{\rho_j} \sum_{k=1}^N a_{kj} x_k (e_k - e_j) + \frac{\partial \ln T}{\partial y} \sum_{j=1}^N x_j e_j \sum_j^T \right] \quad (2.43)$$

If we substitute this expression into (2.16), we find a convenient form of the Stefan - Maxwell expressions for the derivatives of the molar concentrations

$$\frac{1}{x_i} \frac{\partial x_i}{\partial y} = - \frac{\gamma_i}{\rho_i} \sum_{j=1}^N a_{ij} x_j + \sum_{j=1}^N \frac{\gamma_j x_j}{\rho_j} \left[a_{ij} - e_i^* \sum_{k=1}^N (e_k - e_j) a_{kj} x_k \right] + \left[\sum_i^T - e_i \sum_{k=1}^N x_k e_k \sum_k^T \right] \frac{\partial \ln T}{\partial y}, \quad e_i^* = \frac{e_i}{\sum_{k=1}^N x_k e_k^2}, \quad (i = 1, \dots, N) \quad (2.44)$$

Let us now do the same for the mass description of the diffusion. We have

$$\sum_{j=1}^N \tilde{\gamma}_j \frac{\partial \alpha_j}{\partial y} = 0, \quad \tilde{e} = \frac{e}{mc} \\ \frac{E}{kT} = - \left[\sum_{j=1}^N \frac{\gamma_j x_j}{\rho_j} \sum_{k=1}^N x_k a_{kj} (e_k + e_j) + \frac{\partial \ln T}{\partial y} \sum_{j=1}^N x_j e_j \sum_j^T \right] \left(\sum_{k=1}^N x_k e_k^2 \right)^{-1} \quad (2.45)$$

$$\frac{1}{\alpha_i} \frac{\partial \alpha_i}{\partial y} = - \frac{\gamma_i}{\rho_i} \sum_{j=1}^N x_j a_{ij} + \sum_{j=1}^N \frac{\gamma_j x_j}{\rho_j} \left[a_{ij} + \sum_{k=1}^N x_k a_{kj} \left(\frac{m_j}{m} - \frac{m_k}{m} \right) - (e_i^* - \gamma) \sum_{k=1}^N x_k a_{kj} (e_k - e_j) \right] + \left[\sum_i^T - \sum_{k=1}^N c_k \sum_k^T - (e_i^* - \gamma) \sum_{k=1}^N x_k e_k \sum_k^T \right] \frac{\partial \ln T}{\partial y}, \quad \gamma = \frac{\sum_{k=1}^N c_k e_k}{\sum_{k=1}^N x_k e_k^2} \quad (2.46) \quad \underline{17}$$

Thus, the problem was reduced to solving the combined Equations (2.13), (2.14), the equations of diffusion for the reaction products (2.28), and equations of element diffusion (2.33) combined with Expressions (2.44) or (2.46), the heat flux equation (2.18) and Equation (2.22) in the presence of integrals (2.11), (2.26) and (2.27). The equations represent a system of $2N + 4$ equations for $2N + 4$ unknown functions: $u, v, T, \rho, \alpha, \gamma_i$ ($i = 1, \dots, N$).

On the strength of the presence of the integrals one of the diffusion

equations for the reaction products (2.28) can be dropped; and we can also drop the diffusion equation for the electron element from the equations in (2.23). Then the concentration and the diffusion flux of one of the components and the concentration and diffusion flux of the electron component may be found from the above integrals.

If we determine the effective ambipolar coefficients of diffusion in the following manner [9, 10]:

$$\frac{1}{D_i^{(a)}} = \sum_{j=1}^N x_j a_{ij} + c_i \sum_{j=1}^N \left\{ \sum_{k=1}^N c_k a_{kj} \left[\frac{m}{m_j} - \frac{m}{m_k} + (e_i^* - \gamma)(e_k - e_j) \frac{m^2}{m_j m_k} \right] - \frac{m}{m_j} a_{ij} \right\} \frac{\gamma_j}{\gamma_i} \quad (2.47)$$

($i = 1, \dots, N$)

then the Stefan - Maxwell Expressions (2.46) will assume the form of generalized Fick laws

$$\gamma_i = -D_i^{(a)} \frac{\partial c_i}{\partial y} + \rho_i D_i^{(a)} \left[\sum_{l=1}^N c_l - \sum_{k=1}^N c_k \sum_{l=1}^N (e_i^* - \gamma)(e_k - e_l) \frac{m^2}{m_l m_k} \right] \frac{\partial \ln T}{\partial y}. \quad (2.48)$$

On a given field of concentrations, temperature and pressure, the relationships in (2.47) may be studied as a system of N equations for determining $2N$ unknowns: $D_1^{(a)}, \dots, D_N^{(a)}; \gamma_1, \dots, \gamma_N$. This system should be supplemented by the following three equations

$$\sum_{j=1}^N \gamma_j = 0, \quad \sum_{j=1}^N \frac{\gamma_j}{D_j^{(a)}} = 0, \quad \sum_{j=1}^N \frac{e_j}{m_j} \gamma_j = 0 \quad (2.49)$$

For a mixture of only neutral components, this latter relationship from (2.49) should be omitted. The second relationship from (2.49) indicates that the equations in (2.47) are not dependent. Therefore, the independent equations from (2.47) and (2.49) will be $N + 2$ altogether. If we have some solution to this system: $D_1^{(a)}, \dots, D_N^{(a)}; \gamma_1, \dots, \gamma_{N-1}, \gamma_N$, then it is easy to see that the solution will also be: $D_1^{(a)}, \dots, D_N^{(a)}; \alpha \gamma_1, \dots, \alpha \gamma_N$ where α is an arbitrary

number. Therefore, we shall assume as unknown $2N - 1$ values: $D_1^{(n)}, \dots, D_N^{(n)}$, $\gamma_1/\gamma_N, \dots, \gamma_{N+1}/\gamma_N$ for which the System (2.47) and (2.49) will be linear. From the preceding it follows that this system will have a unique solution when and only when $N = 2$ (the case of a neutral mixture), and when $N = 3$ (the case of plasma). In the case of a neutral mixture when $N = 2$, the solution will be:

$$D_1 = D_2 = D_{ij} \quad \frac{\gamma_i}{\gamma_j} = -1 \quad (2.50)$$

In the case of plasma, consisting of three components (I^n , I^{n+1} , E), the solution will be

$$\begin{aligned} \frac{1}{D_n^{(n)}} = \frac{1}{D_{n+1}^{(n)}} = \frac{1}{D_E^{(n)}} &= \frac{\delta_1}{\delta} a(E, n+1) + \frac{\delta_2}{\delta} a(n, E) + \frac{\delta_3}{\delta} a(n, n+1) \\ \delta_1 &= e_n^2 x_n, \quad \delta_2 = e_{n+1}^2 x_{n+1}, \quad \delta_3 = e^2 x_E, \quad \delta = \delta_1 + \delta_2 + \delta_3 \\ \frac{\gamma_n}{\gamma_E} &= -\frac{m_n}{m_E}, \quad \frac{\gamma_{n+1}}{\gamma_E} = \frac{m_{n+1}}{m_E} \end{aligned} \quad (2.51)$$

It is easy to show that the effective coefficients of diffusion in molar description in this particular case will agree respectively with Expressions (2.50) and (2.51). In the case when $N > 2$ (neutral mixture) or $N > 3$ (plasma), System (2.47) and (2.49) has an innumerable set of solutions. In this case to find the unique solution we must have supplemental equations in an amount $N - 2$ for the neutral mixture and $N - 3$ for the plasma. These supplemental expressions may be obtained only after finding the actual solution, when the ratios $\frac{\gamma_1}{\gamma_N}, \dots, \frac{\gamma_k}{\gamma_N}$ become known, where $k = N - 2$ for a mixture of neutral gases and $k = N - 3$ for plasma. See Reference [10] for a more detailed description. Thus, if the diffusion currents can be represented in the form of Fick laws (we can ignore thermal diffusion)

$$J_i = -\rho D_i \frac{\partial c_i}{\partial y} \quad (i = 1, \dots, N) \quad (2.52)$$

then the Lewis - Semenov number must appear in the equations

$$L_i = \frac{\rho c_p k_i}{\lambda} \quad (i=1, \dots, N) \quad (2.53)$$

The diffusion equations (2.15) and the energy equation (2.42) can then be rewritten as:

$$\rho(u \frac{\partial c_i}{\partial x} + v \frac{\partial c_i}{\partial y}) = \frac{\partial}{\partial y} \left(\frac{\mu}{S_i} \frac{\partial c_i}{\partial y} \right) + \dot{W}_i, \quad S_i = \frac{\mu}{\rho k_i} \quad (i=1, \dots, N) \quad (2.54)$$

$$\begin{aligned} & \rho u \frac{\partial}{\partial x} (H - H_e) + \rho v \frac{\partial}{\partial y} (H - H_e) + \sum_{j=N+1}^N \left\{ \rho u \frac{\partial}{\partial x} [(g_j^* - g_e^*) h_j] + \rho v \frac{\partial}{\partial y} [(g_j^* - g_e^*) h_j] + \right. \\ & \left. + \frac{\partial}{\partial y} (j_j^* h_j) \right\} = \frac{\partial}{\partial y} \left\{ \frac{\lambda}{c_p} \left[\frac{\partial}{\partial y} (H - H_e) + (G-1) \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) - \sum_{j=N+1}^N (g_e^* - g_j^*) \frac{\partial h_j}{\partial y} - \sum_{k=1}^{N_e} (L_i - 1) Q_k \frac{\partial c_k}{\partial y} \right] \right\} \end{aligned} \quad (2.55)$$

where S_i is the Schmidt number.

The validity of Equations (2.54) and (2.55) is sometimes postulated without sufficient bases for a multi-component mixture with substantially different diffusion properties of the components. The system of basic laminar boundary layer equations, given above for the stationary two-dimensional motions, assumes for certain assumptions still another series of Crocco integrals besides those given above which are always valid.

Let us assume: (1) the specific heats of the elements are identical; (2) the thermal diffusion and diffusion thermal effect are small ($\partial T = 0$, $(i=1, \dots, N)$; (3) $G = S_i = L_i = 1$ ($i=1, \dots, N$); (4) the flow is "frozen": $\dot{W}_i = 0$ ($i=1, \dots, N$); and; (5) $\frac{dU_e}{dx} \equiv 0$ — the streamline flow is around the plate. Then the following particular integrals are valid:

$$H - H_e = \alpha U + \beta, \quad c_i = \alpha_i U + \beta_i \quad (i=1, \dots, N) \quad (2.56)$$

where α , β , α_i , β_i are constants.

Thus, in this ideal case, it is sufficient to find only the velocities from the system of two equations (2.13) and (2.14) with known variable coefficients ρ and μ , and the problem will be completely solved if the boundary conditions for H and C_1 are compatible with (2.56).

In the general case, significant mathematical difficulties are involved in finding the solution to the system of laminar multi-component boundary layer equations given above with reactions in the current even for particular problems.

In conclusion of this section, let us cite the Stefan - Maxwell expression allowing for barodiffusion. For the quasi-neutral mixture [6], we have

$$\begin{aligned} \frac{1}{\alpha_i} \nabla \alpha_i &= \sum_{j=1}^N x_j a_{ij} \left(\frac{\vec{J}_j}{\rho_j} - \frac{\vec{J}_i}{\rho_i} \right) + \frac{e \vec{E}}{kT} + \nabla \ln T \sum_i^T + \nabla \ln p \left(\frac{m_i}{m} - 1 \right) \\ \frac{1}{\alpha_i} \nabla \alpha_i &= - \frac{\vec{J}_i}{\rho_i} \sum_{j=1}^N x_j a_{ij} + \sum_{j=1}^N \left[a_{ij} - \sum_{k=1}^N x_k a_{kj} \left(\frac{m_k}{m} - \frac{m_j}{m} \right) \right] \frac{x_j \vec{J}_j}{\rho_j} + \\ &+ \left(e_i - \sum_{k=1}^N c_k e_k \right) \frac{\vec{E}}{kT} + \nabla \ln T \left(\sum_i^T - \sum_{k=1}^N c_k \sum_k^T \right) + \nabla \ln p \left(\frac{m_i}{m} - \sum_{k=1}^N \frac{m_k c_k}{m} \right) \end{aligned}$$

From the condition of quasi-neutrality, we find the field \vec{E} :

$$- \frac{\vec{E}}{kT} \sum_{k=1}^N x_k e_k = \sum_{j=1}^N \frac{\vec{J}_j}{\rho_j} a_{ij} \sum_{k=1}^N x_k a_{kj} (e_k - e_j) + \nabla \ln T \sum_{k=1}^N x_k e_k \sum_k^T + \nabla \ln p \sum_{k=1}^N c_k e_k. \quad (1.19)$$

If we substitute this expression into the previous expressions we ultimately find

$$\begin{aligned} \frac{1}{\alpha_i} \nabla \alpha_i &= - \frac{\vec{J}_i}{\rho_i} \sum_{j=1}^N x_j a_{ij} + \sum_{j=1}^N \left[a_{ij} - e_i^* \sum_{k=1}^N x_k a_{kj} (e_k - e_j) \right] \frac{x_j \vec{J}_j}{\rho_j} + \\ &+ \nabla \ln T \left(\sum_i^T - e_i^* \sum_{k=1}^N x_k e_k \sum_k^T \right) + \nabla \ln p \left(\frac{m_i}{m} - 1 - e_i^* \sum_{k=1}^N c_k e_k \right) \quad (2.44') \\ &\quad (i = 1, \dots, N) \end{aligned}$$

$$\begin{aligned}
\frac{1}{\alpha_i} \nabla \alpha_i = & - \frac{\vec{y}_i}{\beta_i} \sum_{j=1}^N x_j \alpha_{ij} + \sum_{j=1}^N \left[\alpha_{ij} - \sum_{k=1}^N \alpha_{ik} \alpha_{kj} \left(\frac{m_k}{m} - \frac{m_j}{m} \right) - \right. \\
& \left. - (e_i^* - \gamma) \sum_{k=1}^N \alpha_{ik} \alpha_{kj} (e_k - e_j) \right] \frac{\vec{y}_j x_j}{\beta_j} + \nabla \ln T \left[\sum_{i=1}^N \alpha_i - \sum_{k=1}^N \alpha_k \sum_{i=1}^N \alpha_{ki} - (e_i^* - \gamma) \sum_{k=1}^N \alpha_{ik} \sum_{j=1}^N \alpha_{kj} \right] \\
& + \nabla \ln p \left[\frac{m_i}{m} - \sum_{k=1}^N \alpha_{ik} \frac{m_k}{m} - (e_i^* - \gamma) \sum_{k=1}^N \alpha_{ik} e_k \right] \quad (i=1, \dots, N)
\end{aligned} \tag{2.46'}$$

$$e_i = \frac{e_i}{\sum_{k=1}^N \alpha_{ik} e_k^2}, \quad \gamma = \frac{\sum_{k=1}^N \alpha_k e_k}{\sum_{k=1}^N \alpha_k e_k^2}$$

These are two equivalent systems of $N - 1$ independent equations each.

2.3 Boundary Conditions.

The boundary conditions at the outermost boundary of the boundary layer are the ordinary ones. The unknown functions must satisfy the asymptotic approach to their values at the outermost boundary of the boundary layer:

$$u(x, y) \rightarrow u_e(x), \quad H(x, y) \rightarrow H_e = \text{const}, \quad \alpha_i \rightarrow \alpha_{ie}(x) \quad (i=1, \dots, N) \tag{2.57}$$

when $y \rightarrow \infty$

The values of the functions $u_e(x), H_e, \alpha_{ie}(x)$ ($i=1, \dots, N$) must be determined from solution to the problem on the motion of a nonviscous liquid.

Furthermore, we shall take advantage of the condition that at the outermost boundary of the boundary layer the concentrations of elements are constant and equal to their values in the approaching current:

$$C_j^*(x, y) \rightarrow C_j^* = C_j^\infty = \text{const} \quad (j=1, \dots, N_{el}) \tag{2.58}$$

The temperature on the outermost boundary of the boundary layer must

satisfy the differential energy equation described along the jet stream of the external nonviscous flow

$$C_{pe} \frac{dT_e}{dx} - \sum_{k=1}^{N_k} Q_k(T_e) \frac{dC_{ke}}{dx} + \frac{d}{dx} \left(\frac{U_e^2}{2} \right) = 0 \quad (2.59)$$

or, which is the same,

$$\sum_{j=N_k+1}^N C_{je}^* \frac{dh_{je}}{dx} - \sum_{k=1}^{N_k} \frac{d}{dx} [Q_k(T_e) C_{ke}] + \frac{d}{dx} \left[\frac{U_e^2}{2} \right] = 0 \quad (2.59')$$

If we solve this latter equation allowing for (2.58), we find

$$\sum_{j=N_k+1}^N C_{je}^* h_{je} - \sum_{k=1}^{N_k} C_{ke} Q_k(T_e) + \frac{U_e^2}{2} = \text{const} = H_e = h_{\infty} + \frac{U_{\infty}^2}{2} \quad (2.60)$$

Allowing for (2.2), we can write

$$h_{\infty} = \sum_{k=1}^N C_{k\infty} h_{k\infty} = \sum_{j=N_k+1}^N C_{j\infty}^* h_{j\infty} - \sum_{k=1}^{N_k} C_{k\infty} Q_k(T_{\infty}) \quad (2.61)$$

Then from (2.60), (2.61) and (2.58) we find the equation for T_e

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$$\sum_{j=N_k+1}^N C_{j\infty}^* \int_{T_{\infty}}^{T_e} C_{pj} dT - \sum_{k=1}^{N_k} [C_{ke} Q_k(T_e) - C_{k\infty} Q_k(T_e)] + \frac{U_e^2 - U_{\infty}^2}{2} = 0 \quad (2.62)$$

Here $U_{\infty}, T_{\infty}, C_{i\infty}$ ($i=1, \dots, N_k$), $C_{j\infty}^*$ ($j=1, \dots, N_k$) are constants given from the conditions of the problem. With external flow around bodies usually: $C_{i\infty}=0$ ($i=1, \dots, N_k$). Thus, unlike the single-component boundary layer, here we must give N concentrations at the outermost boundary of the boundary layer, which greatly complicates formulation of the problem since these data, as a rule, are lacking. The published solutions to problems of the flow around bodies by a nonviscous gas in the presence of reactions usually do not contain complete information on the composition of the gas on the surface.

However, with certain reasonable assumptions relative to the character of the reactions in the jet stream of a nonviscous flow over a body or with additional assumptions relative to the character of the interaction between the hydrodynamic parameters and the chemical reactions, the concentrations $C_{ie}(x)$ can be easily found (see [6]).

Let us note that under the conditions of (2.57) the first and second derivatives of the unknown functions along y strive to zero when $y \rightarrow \infty$. The "initial" conditions along the x coordinate will correspond to the condition of symmetry on the center line of the flow. Quite different boundary conditions may be given on the surface of a body, depending on the type of problem involved.

First let us look at the classical formulation of the problem of finding viscous friction and convective heat flux for a body that is impermeable for all components and which has a given temperature of the surface

$$T(x, 0) = T_w(x) \quad (2.63)$$

$T_w(x)$ is a known function of x .

If the surface of the body is stationary (no walls on it), then in the absolute system of coordinates associated with the body, the conditions of attachment and impermeability will give

$$u(x, 0) = 0 \quad v(x, 0) = 0 \quad (2.64)$$

The conditions for concentrations on the body will be different depending on the physico-chemical properties of the wall. But in any case, for an impermeable wall, the conditions of zero flow of the elements must be satisfied.

$$\gamma_j^*(x, 0) = \gamma_j(x, 0) + \sum_{k=1}^{N_e} \frac{\gamma_{kj} m_j}{m_k} \gamma_k(x, 0) = 0 \quad (j = 1, \dots, N_e = N - N_r) \quad (2.65)$$

These conditions essentially [see (2.46)] superpose the relationship on the derivatives of the concentrations along the normal coordinate on the wall.

The remaining $N_r = N - N_e$ conditions for the concentrations will depend on the character of the reactions on the wall. If the wall is an ideal catalyzer, then the conditions of chemical balance must be satisfied on it. These will often be the reactions of recombination and neutralization. Thus,

$$\prod_{j=N_e+1}^N \left(\frac{C_{jW}}{m_j} \right)^{\gamma_{ij}} \left(\frac{C_{iW}}{m_i} \right)^{-1} = \frac{K_{p,i}(T_w)}{(P \cdot m)_W^{\gamma_i}}, \quad \gamma_i = \sum_{j=N_e+1}^N \gamma_{ij} - 1 \quad (i = 1, \dots, N_r) \quad (2.66)$$

$$\frac{x_w(I_i^{(k)}) x_w(E)}{x_w(I_i^{(k-1)})} = \frac{K_p(I_{iw}^{(k)})}{P(x)} \quad \begin{pmatrix} k = 1, 2, \dots, z_i \\ i = 1, \dots, N_r \end{pmatrix} \quad (2.67)$$

$$z_1 + z_2 + \dots + z_{N_r} = N_r \quad N_{r1} + N_{r2} = N_r$$

There will be exactly N conditions in (2.65) - (2.67).

When the reactions on the wall take place at finite rates, instead of (2.66) and (2.67), it follows that

$$\gamma_i(x, 0) = \dot{r}_i(p, T, C_1, \dots, C_N) \quad (i = 1, \dots, N_r) \quad (2.68)$$

where \dot{r}_i , $[\dot{r}_i] = \text{gram/cm}^2 \text{sec}$ is the surface density of formation of the i -th /21 component due to heterogeneous chemical reactions. Determination of the function \dot{r}_i in each specific case constitutes a fundamental problem. Finally in the case of a chemically neutral wall

$$\gamma_i(x, 0) = 0 \quad (i = 1, \dots, N_r) \quad (2.69)$$

Along with the conditions in (2.65) this is reduced to the following final boundary conditions on a chemically neutral wall

$$\gamma_i(x, 0) = 0 \quad (i = 1, \dots, N) \quad (2.70)$$

Now let us look at the more complex case of boundary conditions on a body when its surface is thermochemically disintegrated, i.e., when the material of the body enters into a physico-chemical interaction with the gases of the boundary layer.

For concreteness let us look at the following model of disintegration which to some degree models the disintegration of heat shield coatings, for example, textolite.

After heating of a semi-infinite body up to a certain characteristic temperature T^* , let disintegration reactions (pyrolysis) of a part of the products begin to take place. As a result of this, gases and porous condensed residue (coke residue) are formed, through which these gases may be filtered and transported through the surface to the boundary layer. With further heating of the body temperatures higher than T^* , the pyrolysis front will be shifted inside the body. The surface of the body acquires a temperature $T_w > T^*$, and heterogeneous reactions may additionally take place on it with pyrolytic gases and gases from the boundary layer escaping from the body. Furthermore, the reactions may occur also between the filtering gases and the residue. There will be two regions in the body then. The first region is a carbonized layer with gases filtering through it, and the second region is a heated body having the initial physical properties. During nonstationary heating the surface of the body and the pyrolysis front will be shifted at different velocities. Under stationary heating conditions, which we shall study, both fronts will be shifted at an identical velocity, equal to the disintegration rate of the body v around which the flow is passing. This velocity will be a function of x .

Therefore, strictly speaking, even for stationary flow conditions the flow over the disrupted surface will be nonstationary. Then in writing the boundary layer equations in the coordinate system associated with the disintegration front as we shall do, terms are added of order $\frac{\partial}{\partial \tau} = \frac{\rho}{\rho^*} \frac{\partial}{\partial \tau^*}$, where $\rho^{(1)}$ is the density of the body. Since $\frac{\rho}{\rho^*} \sim 10^{-3}$, these terms can then be ignored. Therefore, the boundary layer equations in this movable coordinate system remain the same as in the absolute coordinate system associated with the body. Of course, here we shall assume that as a result of disintegrating the surface of the body is sufficiently smooth, i.e., $\alpha L \ll \sqrt{Re}$. Here α is the curvature of the surface, L is the characteristic length, Re is the Reynolds number.

In this case a vertical component of the velocity vector appears on the wall that is non-zero, the so-called flow. We shall assume in this paper that the flow is sufficiently small, so that the sufficient conditions are not disrupted for validity of the boundary layer equations. Here we shall also assume that the distribution of pressure, velocity, temperature and concentrations on the outermost boundary of the boundary layer will be the same as with streamline flow around a non-disintegrating body. Then the conditions for $y \rightarrow \infty$ remain the same, i.e., (2.57), (2.58), (2.62).

We shall also postulate satisfaction of the condition of attachment (2.64).

If we look at the general case of a heterogeneous heating of the body, then the escape of a component will take place both due to surface heating and due to flow through the pores inside the material of the body⁽⁴⁾, i.e.,

(4) This case is typical for the thermochemical disintegration of thermoplastics. For example, with heating of textolite, carbon monoxide is formed on the surface both due to heterogeneous heat of the coke residue and due to the flow of CO, which forms as a result of pyrolysis of the plastic.

$$\rho_i v_i|_{y=0} = \dot{u}_i + \rho_i^{(d)} v_i^{(d)} \quad (i=1, 2, \dots, N) \quad (2.71)$$

where v_i is the mean statistical velocity of the i -th component, $\rho_i^{(d)} v_i^{(d)}$ is the density of the mass current of the flowing part of the i -th component, \dot{u} is the surface density of the formation of the i -th component in the heterogeneous reactions. If we sum the conditions in (2.71) we find, by taking into account that for a flowing mass of gas the law of conservation of mass is valid, that

$$\rho v|_{y=0} = \rho^{(d)} v^{(d)} = \rho^{(d)} \mathcal{D} \quad (2.72)$$

where $\rho^{(d)} v^{(d)}$ is the mass surface velocity of the thermochemical disintegration of the body.

Then (2.71) can be written as:

$$(\rho v)_w (C_i - C_i^{(d)})_w + \mathcal{J}_{iw} = \dot{u}_i + \mathcal{J}_i^{(d)} \quad \mathcal{J}_i^{(d)} = \rho_i^{(d)} (v_i^{(d)} - v^{(d)}) \quad (i=1, \dots, N) \quad (2.73) \quad 122$$

where $\mathcal{J}_i^{(d)}$ is the diffusion flow of the i -th component in the body with approach to the surface from the side of the body.

Under stationary conditions, disintegration from the elementary examinations of the laws of conservation of mass of the components, it will follow that the gases inside the coke layer must be at rest relative to the residue. Then $\mathcal{J}_i^{(d)} = 0 \quad (i=1, \dots, N)$. In the general case determination of the currents $\mathcal{J}_i^{(d)}$ is an independent problem, whose solution will depend on the detailed model of the thermochemical disintegration of the body. In this paper we shall assume the flows $\mathcal{J}_i^{(d)} \quad (i=1, \dots, N)$ equal to zero.

If we multiply Equation (2.73) by $\overline{m_j^* k} = \frac{\partial k_j m_j}{\partial m_k}$ and sum over all components, we then find the conditions for conservation of the elements on the surface

$$(\rho v)_w (c_j^* - c_j^{*(1)}) + \gamma_{jw}^* = 0 \quad (j = 1, \dots, N_e) \quad (2.74)$$

where $C_j^{*(1)}$ is the concentration of elements of the body's material with approach to the heating front from the side of the body. These numbers are specified in this formulation.

Instead of the Conditions (2.73) it is more convenient to use the conditions of conservation of elements (2.74) and $N - N_e$ in the solution. These are conditions of conservation of mass in the independent N_r reactions

$$(\rho v)_w (c_l - c_l^{(1)})_w + \gamma_{lw} = \dot{w}_l \quad (l = 1, \dots, N_r) \quad (2.75)$$

With a specified value of $(\rho v)_w$ Expressions (2.74) and (2.75) give exactly N conditions for N concentrations.

In the case of chemical balance on the surface, Conditions (2.75) must be replaced by Expressions (2.66), (2.67). Conditions (2.74) remain.

In the specified formulation the temperature on the wall is not given, therefore, we must turn our attention to the condition of conservation of energy at the heating front which in approximation of the boundary layer theory will be [11] (the index "w" is omitted)

$$\rho v h + q + \sum_{k=1}^N h_k \gamma_k - q^R + \epsilon T^4 \sigma R = \rho^{(1)} v^{(1)} h^{(1)} + q^{(1)}, \quad \rho v = \rho^{(1)} v^{(1)} \quad (2.76)$$

The asymmetry of condition (2.76) is the result of the assumption that the material of the streamlined body does not transmit and does not absorb the radiation, and no diffusion phenomenon appears in it.

Since a part of the body components are in the condensed phase, then

$$\tilde{h}_i = h_i^{(g)} - \Delta_i^\phi \quad (i=1, \dots, N) \quad \tilde{h}^{(g)} = \sum_{k=1}^N c_k^{(g)} \tilde{h}_k \quad (2.77)$$

where $h_i^{(g)}$ is the enthalpy of the i -th component in the gaseous state at ϕ the surface temperature of the body, Δ_i^ϕ is the heat of the phase transition of the i -th component from the condensed state to the gaseous state.

If we use (2.77), (2.2) and (2.74), Condition (2.76) may be written as

$$-\sum_{k=1}^{N_k} Q_k (\rho v c_k + \tilde{h}_k) + q - q^R + \epsilon \sigma R T^4 = -\sum_{k=1}^{N_k} Q_k (\rho^{(g)} v^{(g)} c_k^{(g)}) + q^{(g)} - \rho v \Delta^\phi \quad (2.76)$$

$$\Delta^\phi = \sum_{k=1}^N c_k \Delta_k^\phi \quad (2.78)$$

where Δ^ϕ is the total heat of the phase transition.

It is easy to show that under stationary disintegration conditions [12]

$$q^{(g)} = -\rho^{(g)} v^{(g)} (\tilde{h}^{(g)} - \tilde{h}_{-\infty}) \quad (2.79)$$

where $\tilde{h}^{(g)} - \tilde{h}_{-\infty}^{(g)}$ is the heat necessary for heating a unit of mass of the body material from the initial temperature $T_{-\infty}$ inside the body up to the temperature on the surface by allowing for all the possible pyrolytic reactions excluding the phase transitions at the front.

Finally the condition of conservation of energy on the surface of a body which is disintegrating and heated from the surface is written as

$$-\sum_{k=1}^{N_k} Q_k \tilde{h}_k + q - q^R + \epsilon \sigma R T^4 + \rho v \Delta = 0 \quad (2.80) \quad \underline{123}$$

where

$$\Delta = \Delta_{\tau}^{\phi} - \sum_{k=1}^{N_k} Q_k (C_{kw} - C_k^{(h)}) + \tilde{h}^{(h)} - \tilde{h}_{-\infty} \quad (2.81)$$

is the total heat necessary for heating a unit of mass of the body material from the initial temperature $T_{-\infty}$ to the temperature on the surface by allowing for the absorption of heat in the body under all possible pyrolytic reactions (the term $\tilde{h}^{(1)} - \tilde{h}_{-\infty}$), in all phase transitions (the term Δ^{ϕ}), in all heterogeneous reactions at the front up to formation of the gas composition on the surface equal to C_{iw} ($i = 1, \dots, N_r^*$).

We should make some comment relative to q^R . If the gas on the boundary layer is transparent, then q^R may be taken from the solution to the flow problem around a body by a nonviscous gas, taking radiation into account. However, in the presence of a flow of a foreign gas and absorption of the radiation in it, we can show that q^R will differ significantly from the radiation q_e^R which is incident on the outermost boundary of the boundary layer, i.e.,

$$q^R \equiv q_w^R = q_e^R - \gamma_R(\rho v)_w \quad (2.82)$$

where the value γ^R is a complex function of interaction between the radiation and the flow in the boundary layer. It may be determined only after solving the problem of flow in the boundary layer allowing for the radiation. This problem is of independent interest and has not been studied here. In this respect let us mention that the flows \mathcal{J}_i ($i = 1, \dots, N_r$) and q in (2.80) also must be found by allowing for the interaction between the flow and the radiation field. However, we can assume that with these values of the flow for which q^R becomes significantly different from q_e^R , the flows \mathcal{J}_i and q will be much less than q^R , and this will not lead to any notable error in the equation of energy balance (2.80). Finally, if \mathcal{J}_i ($i = 1, \dots, N$) and q are determined from System (2.13) - (2.22), then (2.80) will hold only in the case if the radiation does not interact with the flow field. Generally (2.80) is an exact expression. An unknown function $(\rho v)_w$ enters in the

conditions computed above. Therefore, in the problems of thermochemical disintegration we must, in addition to the laws of conservation, add the conditions for continuity of the tangential velocity component and temperature. The supplemental condition must follow from the specific form of the disintegration mechanism. For example, in the case of pure evaporation this may be either the condition of balanced evaporation or the condition of evaporation at a finite rate [13]. In the case of disintegration of complex materials the missing condition may be taken from experiment in the form of a kinetic curve, which relates the disintegration rate and the temperature of the surface [6], etc.

Below we shall examine in greater detail the basic formulations of the problems.

§ 3. PROBLEM ON A CHEMICALLY FROZEN FLOW IN A BOUNDARY LAYER OVER AN IMPERMEABLE SURFACE HAVING ARBITRARY CATALYTIC CHARACTERISTICS

If the flow in a boundary layer is frozen, i.e., the reactions take place so slowly that they can not take place during the life time of the liquid particle in the boundary layer, we can then use W_i ($i = 1, \dots, N$) in all the diffusion equations in (2.15). From these equations it will follow of necessity that $C_{ie} = \text{const}$ when $y \rightarrow \infty$. That is to say, for a "frozen" flow in a boundary layer, we must of necessity look at the "frozen" flow in a non-viscous flow over the body. Furthermore, it often makes sense to consider the "frozen" flow in the boundary layer as a flow which simulates the balanced flow in it, for ease of solution. In fact, if the distribution of concentrations (composition) does not strongly influence the transport coefficients, the difference in the chemically balanced flow from the "frozen" flow with heterogeneous balanced reactions on the wall will consist of the fact that in this latter case all the reactions take place at the wall, and in a "balanced" flow they are distributed along the layer, which often does not change the final result greatly. We can sometimes replace the equili-

brium flow over a "cold" wall by a frozen flow over an ideally catalytic wall. In this case, we must of necessity set $C_{ie} = \text{const}$ ($i = 1, \dots, N$) at the outermost boundary of the boundary layer when $x > x_0$, from which we regard the flow as frozen.

For such a model of the chemical reactions, in the previous equations, we must set $N_r = 0$ and the problem of determining the thermal flux and viscous friction for $x > x_0$ will be reduced to a combined solution of Equations (2.13), (2.14), and the diffusion equations

$$\rho(u \frac{\partial c_i}{\partial x} + v \frac{\partial c_i}{\partial y}) + \frac{\partial \gamma_i}{\partial y} = 0 \quad (i=1, \dots, N) \quad (3.1)$$

along with the Expressions in (2.46) and the energy equations

$$\begin{aligned} \rho u \frac{\partial H^T}{\partial x} + \rho v \frac{\partial H^T}{\partial y} + \sum_{k=1}^N \left\{ \rho u \frac{\partial}{\partial x} [(c_k - c_{ke}) h_k] + \rho v \frac{\partial}{\partial y} [(c_k - c_{ke}) h_k] + \frac{\partial}{\partial y} (\gamma_k h_k) \right\} = \\ = \frac{\partial}{\partial y} \left[-q + \mu \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) \right] \quad H^T = \sum_{k=1}^N h_k c_{ke} + \frac{u^2}{2} \end{aligned} \quad (3.2)$$

or the heat flux equations (2.18) in which we must set $\dot{W}_i = 0$ ($i=1, \dots, N$).

The boundary conditions on the outermost boundary of the boundary layer will be

$$u(x, y) \rightarrow u_e(x), \quad H^T(x, y) \rightarrow H_e^T = \text{const}, \quad c_i(x, y) \rightarrow c_{ie} = \text{const} \quad (3.3)$$

when $y \rightarrow \infty$

Under such conditions Equations (3.1), (3.2) will be consistent with these boundary conditions.

If the wall is impermeable and is maintained at a temperature $T_w(x)$, then boundary conditions (2.65) - (2.68) must be satisfied on it; these conditions in the case of an ideally catalytic wall will be reduced to (2.65) and (2.66), (2.67). Furthermore, we must give the "initial" conditions when $x = x_0$:

$$u(x_0, y) = u_0(y), \quad H^T(x_0, y) = H_0^T(y), \quad G_i(x_0, y) = G_{i0}(y) \quad (i=1, \dots, N) \quad (3.4)$$

which become known after solving the problem of the boundary layer when $x < x_0$.

After solving this problem, we can find the distribution of viscous friction stress along the body from the expression

$$\tau_w = \mu_w \frac{\partial u(x, y)}{\partial y} \quad (3.5)$$

if we ignore the influence of the chemical reactions on the tensor of the viscous stresses.

The total convective heat flux on the catalytic wall will be computed from Expression (2.21), which — by taking into account (2.2), (2.65) and (2.19) — will be (the index "w" is omitted)

$$\begin{aligned} (q_y)_w &= -\lambda \frac{\partial T}{\partial y} - \sum_{k=1}^{N_k} Q_{kw} \gamma_k + R_A T \sum_{k=1}^N \sum_{j=1}^N \frac{x_j}{n_k} \frac{\partial k}{\partial y} \frac{a_{kj}}{m_k} (V_k - V_j) = \\ &= -\lambda \frac{\partial T}{\partial y} - \sum_{k=1}^{N_k} (Q_{kw} + G_k^T \frac{R_A T}{m_k}) \gamma_k = -\lambda \frac{\partial T}{\partial y} - \sum_{k=1}^{N_k} Q_{kw}' \gamma_k \end{aligned} \quad (3.6)$$

where

$$G_i^T = \sum_i^T - \sum_{j=N_k+1}^N v_{ij} \sum_j^T, \quad Q_i' = Q_i + G_i^T \frac{R_A T}{m_i}, \quad (i=1, \dots, N_k) \quad (3.7)$$

For a binary mixture, formed by one reaction,

$$A_k = \kappa A, \quad A_k \equiv i, \quad A \equiv j, \quad m_k = \kappa m_j$$

Accordingly, we find

$$\sum_i^T = -\frac{a_{ij} D_i^T}{\kappa n m_j}, \quad G_i^T = -\frac{a_{ij} D_i^T}{n a_{ij} m_j} \quad (3.8)$$

$$\sigma_i^T \frac{RAT}{m_i} = -\alpha \frac{m^2}{m_i m_j} \cdot \frac{RAT}{m}, \quad \alpha = \frac{K_T}{\alpha_i \alpha_j}, \quad K_T = \frac{m^2}{m_i m_j} \frac{\partial^2 \alpha_{ij}}{\partial^2} \quad (3.9)$$

where the thermodiffusion factor α has a value on the order of 0.1 - 1.

The total heat flux will be equal to

$$(J_q)_w = -\lambda \frac{\partial T}{\partial y} - \left(Q_{iw} - \alpha \frac{m^2}{m_i m_j} \frac{RAT}{m} \right) J_i \quad (3.10)$$

From (3.10) it is easy to evaluate the direct contribution to the thermal flux of the diffusion thermoeffect. The heat of dissipation Q for air is on the order of 7000 cal/gram. Then $\alpha RAT/m < \frac{1}{15} T_w \alpha \approx 20$ cal/gram, when $T_w = 300^\circ$. Therefore, the direct contribution of the diffusion thermoeffect to the heat flux on the wall will be significant (on the order of several percents) only for a sufficiently hot wall ($T_w \sim 20000 - 30000^\circ$ K) and when $\alpha \approx 1$. Since this effect is proportional to the absolute temperature, its influence will be /25 more significant in the hot regions of the boundary layer. This may lead to a significant change in the temperature gradients and the diffusion flows, but to a weak change in them at the wall. Quantitatively this question in an ionized boundary layer has yet to be studied. In the absence of reliable data on the coefficients of thermal diffusion in ionized gases, it is impossible to obtain a final quantitative answer. If the heat flux equation is solved for the function H^T , then it is necessary to convert Expression (3.6) to this function.

We have

$$\lambda \frac{\partial T}{\partial y} = \frac{\lambda}{c_p^*} \frac{\partial H^T}{\partial y} = \frac{\lambda}{c_p} \frac{\partial H^T}{\partial y} + \lambda \left(\frac{1}{c_p^*} - \frac{1}{c_p} \right) \frac{\partial H^T}{\partial y} = \frac{\lambda}{c_p} \cdot \frac{c_p}{c_p^*} \frac{\partial H^T}{\partial y} \quad (3.11)$$

where

$$c_p^* = \sum_{k=1}^N C_{ke} C_{pk} \quad (3.12)$$

Then

$$(\mathcal{J}_q)_w = -\frac{\lambda}{c_p} \cdot \frac{c_p}{c_p^*} \frac{\partial H^T}{\partial y} - \sum_{k=1}^{N_k} Q_{kw}' \mathcal{J}_k \quad (3.13)$$

Let us introduce the relative value for the function H^T .

$$g^T = \frac{H_e^T - H_w^T}{H_e^T - H_w^T} \quad \text{and} \quad g = c_{pw} \frac{T - T_w}{H_e^T - H_w^T} \quad (3.14)$$

and also the relative mass diffusion currents on the wall

$$\mathcal{J}_{iw}' = \frac{\mathcal{J}_{iw}}{c_{ie} - c_{iw}} \quad (i=1, \dots, N) \quad (3.15)$$

Here the value

$$\Delta H^T = H_e^T - H_w^T = \sum_{k=1}^N c_{ke} (h_{ke} - h_{kw}) + \frac{u_e^2}{2} \quad (3.16)$$

is comprised of the heat which is released by one gram of the external current having a composition C_{ie} ($i = 1, \dots, N$) with cooling from a temperature of T_e to the wall temperature of T_w in the absence of chemical reactions and kinetic energy of a nonviscous flow on the wall.

Then (3.13) will be

$$\begin{aligned} (\mathcal{J}_q)_w &= -\left(\frac{\lambda}{c_p^*} \frac{\partial g^T}{\partial y}\right)_w \left[\Delta H^T + \frac{c_p}{\lambda} \sum_{k=1}^{N_k} Q_{kw}' (c_{ke} - c_{kw}) \mathcal{J}_k' \left(\frac{\partial g^T}{\partial y}\right)_w^{-1} \right]_w = \\ &= -\left(\frac{\lambda}{c_p} \frac{\partial g^T}{\partial y}\right)_w \left[\frac{c_p}{c_p^*} \Delta H^T + \frac{c_p}{\lambda} \sum_{k=1}^{N_k} Q_{kw}' (c_{ke} - c_{kw}) \mathcal{J}_k' \left(\frac{\partial g^T}{\partial y}\right)_w^{-1} \right]_w \end{aligned} \quad (3.17)$$

If we introduce the function g from (3.14), then the formula for $(\mathcal{J}_q)_w$ will differ from (3.17) (the second formula) only in that g^T must be replaced by g .

In Formula (3.17) let us introduce the total enthalpy of the braking of the oncoming flow (2.37). Since in the cross section $x = x_0$, where the reactions end and the frozen flow begins, we can use (2.2), then

$$\begin{aligned} \Delta H^T &= \sum_{k=1}^N c_{ke} (h_{ke} - h_{kw}) + \frac{U_e^2}{2} = \sum_{j=N_e+1}^N g_e^* (h_{je} - h_{jw}) - \sum_{k=1}^{N_k} c_{ke} (Q_{ke} - Q_{kw}) + \frac{U_e^2}{2} \\ &= H_e - H_w + \sum_{k=1}^{N_k} (c_{ke} - c_{kw}) Q_{kw} = \Delta H + \sum_{k=1}^{N_k} Q_{kw} (c_{ke} - c_{kw}) \end{aligned} \quad (3.18)$$

Then

$$\begin{aligned} (Y_g)_w &= - \left(\frac{\lambda}{c_p^*} \frac{\partial q^T}{\partial y} \right)_w \left[1 + \sum_{k=1}^{N_k} \frac{Q_{kw}}{Q_{kw}} \left(Y_k' \frac{c_p^*}{\lambda} \left(\frac{\partial q^T}{\partial y} \right)^{-1} + 1 \right) \frac{(c_{ke} - c_{kw}) Q_{kw}}{\Delta H} \right] \Delta H \\ &= - \left(\frac{\lambda}{c_p} \frac{\partial q^T}{\partial y} \right)_w \left[1 + \varepsilon + \sum_{k=1}^{N_k} \frac{Q_{kw}}{Q_{kw}} \left(Y_k' \frac{c_p}{\lambda} \left(\frac{\partial q^T}{\partial y} \right)^{-1} + 1 \right) \frac{(c_{ke} - c_{kw}) Q_{kw}}{\Delta H} \right] \Delta H \end{aligned} \quad (3.19) \quad \underline{126}$$

where

$$\varepsilon = \left(\frac{c_{pw}}{c_p^*} - 1 \right) \left(1 + \sum_{k=1}^{N_k} \frac{(c_{ke} - c_{kw}) Q_{kw}}{\Delta H} \right) \quad (3.20)$$

If we take advantage of the concept of the effective diffusion coefficients introduced for the chemically frozen flows in multi-component boundary layers in References [9, 10] and introduce the relative concentrations

$$S_i = \frac{C_i - C_{iw}}{C_{ie} - C_{iw}} \quad (i = 1, \dots, N) \quad (3.21)$$

$$\begin{aligned} Y_{iw}' \left(\frac{\partial q^T}{\partial y} \right)^{-1} \frac{c_{pw}}{\lambda_w} &= -L_{iw}^{x_i}, \quad L_{ie} = \frac{\rho c_p D_i}{\lambda} \quad (i = 1, \dots, N) \\ Y_{iw}' \left(\frac{\partial q^T}{\partial y} \right)_w^{-1} \frac{c_p^*}{\lambda_w} &= -L_{iw}^{*x_i}, \quad L_{ie}^* = \frac{\rho c_p^* D_i}{\lambda} \quad (i = 1, \dots, N) \end{aligned} \quad (3.22)$$

where the exponents x_i must be determined from the actual solution to the problem, and the effective diffusion coefficients D_i are virtually independent of the exponents x_i and are determined by the boundary values of the concentrations and by the binary diffusion coefficients of the components.

Then

$$\begin{aligned}
 (j_q)_w &= - \left(\frac{\mu}{\sigma^*} \frac{\partial q^T}{\partial y} \right)_w \left[1 - \sum_{k=1}^{N_r} \frac{Q_{kw}'}{Q_{kw}} (L_{kw}^{*z_k} - 1) \frac{(C_{ke} - C_{kw}) Q_{kw}}{\Delta H} \right] \Delta H = \\
 &= - \left(\frac{\mu}{\sigma} \frac{\partial q^T}{\partial y} \right)_w \left[1 + \varepsilon - \sum_{k=1}^{N_r} \frac{Q_{kw}'}{Q_{kw}} (L_{kw}^{z_k} - 1) \frac{(C_{ke} - C_{kw}) Q_{kw}}{\Delta H} \right] \Delta H
 \end{aligned}
 \tag{3.23}$$

where

$$\sigma^* = \frac{\mu c_p^*}{\lambda}, \quad \sigma = \frac{\mu c_p}{\lambda}
 \tag{3.24}$$

Formulas (3.23) are exact structural formulas for the total specific convective heat flux which generalize the known structural formulas of Fay and Riddel [14] and Lees [15] to the case of nonuniform diffusion and thermodynamic properties of the components, and to the nonuniformity of their specific heats. They also take into account the dependence of the specific heats and the heats of reactions on temperature.

The formulas in (3.23) are valid both for an ideally catalytic wall and for a wall with a finite catalytic character. In the latter case the concentrations of the reaction products on the wall C_{iw} ($i = 1, \dots, N_r$) must be known from solving the problem with kinetic boundary conditions (2.68). If we use the conditions in (2.68), Formula (3.13) can be assigned another form

$$(j_q)_w = - \left(\frac{\lambda}{c_p^*} \frac{\partial q^T}{\partial y} \right)_w \left[\Delta H^T + \frac{c_p}{\lambda} \sum_{k=1}^{N_r} Q_{kw}' \left(\frac{\partial q^T}{\partial y} \right)^{-1} z_k \right]
 \tag{3.25}$$

However Formula (3.25) is less clear than (3.23).

§ 4. PROBLEM ON A CHEMICALLY FROZEN FLOW IN A BOUNDARY LAYER OVER A THERMOCHEMICALLY DISINTEGRATING SURFACE POSSESSING AN ARBITRARY CATALYTIC CHARACTER

Here we look at the same scheme of chemical reactions in the flow as in the previous section. Then this problem will be reduced to the combined

solution of the system of Equations (2.13), (2.14), (3.1) and (3.2) with the boundary and "initial" conditions (3.3), (2.74), (2.75), (2.80), (3.4).

After solving this problem, we find the viscous friction stress from Formula (3.5). Instead of the thermal flux, here we must find the mass transport velocity from the surface $(\rho v)_w$ and the temperature of the surface T_w .

This heat flux on the wall q , which appears in (2.80), according to the definition of (2.19) and the boundary conditions in (2.74) may be represented in the form (the index "w" is dropped)

$$\begin{aligned} q_w &= -\lambda \frac{\partial T}{\partial y} - \sum_{k=1}^{N_k} G_k^T \frac{RAT}{m_k} \gamma_k + (\rho v) \sum_{k=N_k+1}^N \sum_k^T \frac{RAT}{m_k} \Delta C_k^* = \\ &= -\frac{\lambda}{c_p^*} \frac{\partial H^T}{\partial y} - \sum_{k=1}^{N_k} G_k^T \frac{RAT}{m_k} (\gamma_k + \rho v \Delta C_k) + \rho v \sum_{k=1}^N \frac{RAT \Delta C_k}{m_k} \sum_k^T \quad (4.1) \end{aligned} \quad 127$$

$$\Delta C_j^* = C_j^* - C_j^{*(1)} \quad (j = N_k+1, \dots, N) \quad \Delta C_i = C_w - C_i^{(1)} \quad (i = 1, \dots, N)$$

If Expressions (4.1) and (4.2) are substituted in (2.80), we find

$$-\sum_{k=1}^{N_k} Q_k' \gamma_k - \frac{\lambda}{c_p^*} \frac{\partial H^T}{\partial y} - q_e^R + \epsilon G_R T^4 + \rho v (\Delta' + \gamma_R) = 0 \quad (4.2)$$

where

$$\begin{aligned} Q_i' &= Q_i + RAT \frac{G_i^T}{m_i} \quad (i = 1, \dots, N_k) \\ \Delta' &= \tilde{h}^{(0)} - \tilde{h}_{-\infty} + \sum_{k=1}^N [C_k^{(0)} \Delta_k + \frac{RAT}{m_k} \sum_k^T (C_k - C_k^{(0)})] - \sum_{k=1}^{N_k} (Q_k + RAT \frac{G_k^T}{m_k}) (C_k - C_k^{(0)}) = \\ &= \Delta + \sum_{k=1}^N \frac{RAT}{m_k} \sum_k^T (C_k - C_k^{(0)}) - \sum_{k=1}^{N_k} RAT \frac{G_k^T}{m_k} (C_k - C_k^{(0)}) \quad (4.3) \end{aligned}$$

It is interesting to note that allowing for the diffusion thermal effect at the disintegration front will finally lead to a slight known change in the reaction heats and the heats of the phase transitions, i.e., if we introduce the concept of effective reaction heats Q_i' and the effective heat of the "phase" transition Δ' , which will depend in a known manner on the thermo-diffusion properties of the components and their concentrations on the wall,

then the energy balance equation on the wall, by allowing for the diffusion thermal effect, will be written in a form which agrees with the writing of this expression without allowing for the diffusion thermal effect. Expression (4.2) for known flows \mathcal{J}_i ($i = 1, \dots, N$) and composition of the gases on the wall gives the relationship between the temperature of the wall T_w and the mass transport velocity $(\rho v)_w$ from the wall. In the process of solving the problem it participates as one of the boundary conditions along with the conditions in (2.74), (2.75). However, if we make certain assumptions relative to the currents \mathcal{J}_i ($i = 1, \dots, N$) and the derivative $\partial H^T / \partial y$ on the wall as a function of the flow, following from the solutions to the individual particular problems, then from (4.2) we can find an effective structural expression for the mass transport velocity.

$$C_{HT} = \frac{(\lambda \frac{\partial T}{\partial y})_w}{\rho_e U_e (H_e^T - H_w^T)} = \frac{(-\frac{M}{G^*} \frac{\partial H^T}{\partial y})_w}{\rho_e U_e (H_e^T - H_w^T)} \quad (4.4)$$

and the concept of coefficients of mass exchange

$$C_i = \frac{-\mathcal{J}_{iw}}{\rho_e U_e (C_{ie} - C_{iw})} \quad (i = 1, \dots, N) \quad (4.5)$$

Let us assume that these coefficients of the heat and mass exchange can be written in the form

$$C_{HT} = C_{HT}^0 - \frac{(\rho v)_w}{\rho_e U_e} \gamma_{HT}, \quad C_i = C_i^0 - \frac{(\rho v)_w}{\rho_e U_e} \gamma_i \quad (i = 1, \dots, N) \quad (4.6)$$

Then by using these representations, Expression (4.2) can be solved for $(\rho v)_w$ and can be written in two equivalent forms

$$(\rho v)_w = \frac{q_e^R - \epsilon G_R T_w^4 + C_{HT}^0 \rho_e U_e [\Delta H^T - \sum_{k=1}^{N_2} \frac{C_k^0}{C_{HT}^0} (C_{ke} - C_{kw}) Q_{k'w}]}{\Delta I + \gamma_R + \gamma_{HT} [\Delta H^T - \sum_{k=1}^{N_2} [C_{ke} - C_{kw}] \frac{\gamma_k}{\gamma_{HT}} Q_{k'}]} = \quad (4.7)$$

$$= \frac{q_e - \varepsilon \sigma_R T_w^4 + C_{HT}^0 p_e U_e \left[\Delta H^T - \sum_{k=1}^{N_k} \frac{C_k}{C_{HT}} (c_{ke} - c_{kw}) Q_{kw}' \right]}{\Delta' + \gamma_R + \gamma_{HT} \left[\Delta H^T - \sum_{k=1}^{N_k} \frac{C_k}{C_{HT}} (c_{ke} - c_{kw}) Q_{kw}' \right]} \quad (4.8)$$

The second expression is more convenient, when the ratios of the mass exchange coefficients to the coefficient C_{HT} vary weakly from the conditions of the problem.

In (4.7) and (4.8) let us introduce the total enthalpy of deceleration of the nonviscous flow, according to (3.18). Then

$$(\rho v)_w = \frac{q_e - \varepsilon \sigma_R T_w^4 + p_e U_e C_{HT} \left[\Delta H - \sum_{k=1}^{N_k} \left(\frac{C_k^0}{C_{HT}} \frac{Q_k'}{Q_k} - 1 \right) (c_{ke} - c_{kw}) Q_k \right]}{\Delta' + \gamma_R + \gamma_{HT} \left[\Delta H - \sum_{k=1}^{N_k} \left(\frac{\gamma_k Q_k'}{\gamma_{HT} Q_k} - 1 \right) (c_{ke} - c_{kw}) Q_k \right]} \quad (4.9) \quad \underline{128}$$

$$= \frac{q_e^p - \varepsilon \sigma_R T_w^4 + p_e U_e C_{HT} \left[\Delta H - \sum_{k=1}^{N_k} \left(\frac{C_k Q_k'}{C_{HT} Q_k} - 1 \right) (c_{ke} - c_{kw}) Q_k \right]}{\Delta' + \gamma_R + \gamma_{HT} \left[\Delta H - \sum_{k=1}^{N_k} \left(\frac{C_k Q_k'}{C_{HT} Q_k} - 1 \right) (c_{ke} - c_{kw}) Q_k \right]} \quad (4.10)$$

When the linear approximation (4.6) is valid for the coefficients of heat and mass exchange, the values γ_{HT} and γ_i ($i = 1, \dots, N_r$) do not depend on $(\rho v)_w$ or on the blast⁽⁵⁾. For large blasts γ_{HT} and γ_i ($i = 1, \dots, N_r$) will depend on the blast, but more weakly than the coefficients of heat and mass exchange themselves. In any case for known γ_{HT} and γ_i ($i = 1, \dots, N_r$) and γ_R , Expressions (4.9), (4.10) are exact structural formulas for the mass transport velocity of a material from the body surface during thermochemical stationary disintegration when the absorption and transmission of the radiation in the body may be ignored. Although from Formulas (4.9) and (4.10) it is impossible to determine the mass transport velocity prior to the actual solving of the problem, nevertheless they are useful for evaluating $(\rho v)_w$. In fact, if for the numerical values of γ_R , γ_{HT} , γ_i

(5) For sufficiently small flows the linear approximation (4.6) is valid.

($i = 1, \dots, N_r$), $C_H T^0$, C_i ($i = 1, \dots, N$), we take those values obtained from solving the simpler problems, then with a specified pressure and composition on the wall, which can be determined approximately, Formulas (4.9) and (4.10) will give a value of $(\rho v)_w$ as a function of the temperature at the front T_w . In fact, however, to compute $(\rho v)_w$, we often use the concept of effective disintegration enthalpy which will be introduced in the next section.

§ 5. DETERMINATION OF THE EFFECTIVE ENTHALPY OF DISINTEGRATION AND ITS EXACT STRUCTURAL FORMULA FOR A CHEMICALLY FROZEN FLOW IN A BOUNDARY LAYER

Let us determine the effective enthalpy of disintegration H_{ef} in the following manner

$$H_{ef} = \frac{(\dot{q})_w}{(\rho v)_w} \quad (5.1)$$

i.e., H_{ef} is equal to the ratio of the total convective heat flux to the nondisintegrating surface, having the same temperature and shape as the disintegrating surface to the mass transport velocity.*

In (3.19) let us introduce the coefficients (4.4) and (4.5). Then

$$(\dot{q})_w = -\rho_e U_e C_H T^0 \left[1 - \sum_{k=1}^{N_r} \left(\frac{C_k^0 Q_{kw}'}{C_H T^0 Q_{kw}} - 1 \right) \frac{(Q_{ee} - C_{kw}) Q_{kw}}{\Delta H} \right]^{(0)} \Delta H \quad (5.2)$$

The symbol (0) over the brackets indicates that the expression must be computed for the mixture of gases forming with flow around the body without allowing for its disintegration, i.e., the sum from 1 to N_r ; for example, in (4.9) or (4.10) there will be contained terms which correspond to the products of burning of the material of the body, which will be absent in

* Translator's Note: There is apparently a mistake in the original text as the meaning is not clear.

the sum in Expression (5.2). Thus, the heat flux (5.2) for this problem of disintegration is an imaginary value. Allowing for (4.9), (4.10) and (5.2), we find

$$H_{ef} = \Delta' + \gamma_R + \gamma_{HT} \Delta H \left[1 - \sum_{k=1}^{N_k} \left(\frac{\gamma_k Q_k'}{\gamma_{HT} Q_k} - 1 \right) \frac{(c_{ke} - c_{kw}) Q_k}{\Delta H} \right] E = \quad (5.3)$$

$$= \Delta' + \gamma_R + \gamma_{HT} \Delta H \left[1 - \sum_{k=1}^{N_k} \left(\frac{C_k Q_k'}{C_{HT} Q_k} - 1 \right) \frac{(c_{ke} - c_{kw}) Q_k}{\Delta H} \right] E \quad (5.4)$$

where

$$E = \frac{\left[1 - \sum_{k=1}^{N_k} \left(\frac{C_k^0 Q_k'}{C_{HT}^0 Q_k} - 1 \right) \frac{(c_{ke} - c_{kw}) Q_k}{\Delta H} \right]^{(0)}}{1 - \sum_{k=1}^{N_k} \left(\frac{C_k^0 Q_k'}{C_{HT}^0 Q_k} - 1 \right) \frac{(c_{ke} - c_{kw}) Q_k}{\Delta H} + \frac{q_e^R - \epsilon \sigma_R T^4}{\rho_e u_e C_{HT} \Delta H}} \quad (5.5)$$

If we use the concept of effective diffusion coefficients and the generalized 29 analogy (3.22), then we will have

$$\frac{C_i^0}{C_{HT}^0} = - \frac{\gamma_{LW} \Delta H^T}{\Delta c \frac{\lambda}{c_p} \frac{\partial H^T}{\partial y}} = L_{LW}^{x_i^0}, \quad \frac{Q}{C_{HT}} = L_{LW}^{x_l} \quad (5.6)$$

If $x \approx x_i^0$, then $\frac{\gamma_L}{\gamma_{HT}} \approx 1$. If we approximately set $0_i^1 \approx 0_i$ ($i = 1, \dots, N_r$) we find

$$H_{ef} = \Delta' + \gamma_R + \gamma_{HT} \Delta H \left[1 - \sum_{k=1}^{N_k} (L_{LW}^{x_k} - 1) (c_{ke} - c_{kw}) \frac{Q_k}{\Delta H} \right] E \quad (5.7)$$

where

$$E = \frac{\left[1 - \sum_{k=1}^{N_k} (L_{LW}^{x_k^0} - 1) \frac{(c_{ke} - c_{kw}) Q_k}{\Delta H} \right]^0}{1 - \sum_{k=1}^{N_k} (L_{LW}^{x_k} - 1) \frac{(c_{ke} - c_{kw}) Q_k}{\Delta H} + \frac{q_e^R - \epsilon \sigma_R T^4}{\rho_e u_e C_{HT} \Delta H}} \quad (5.8)$$

With intense disintegration, when the blast is so great that we can ignore

the convective and diffusion heat transports⁽⁶⁾, from (4.2) we find

$$(\rho v)_w = \frac{q_e^R - \epsilon \sigma_R T^4}{\Delta' + \gamma_R}, \quad \Delta' = \tilde{h}^{(d)} - \tilde{h}_{-\infty} + \sum_{k=1}^N C_k^{(1)} \Delta_k^P \quad (5.9)$$

where the value γ_R is given by approximation (2.82), i.e., equal to

$$\gamma_R = \frac{q_e^R - q_w^R}{(\rho v)_w}$$

and is the heat of the radiation energy, absorbed per unit of mass of the flowing material, of the disintegrating surface. Let us call this the effective heat of absorption of the radiation energy with flow. Then with intense disintegration

$$H_{\text{eff}} = \frac{(\gamma_R)_w}{q_e^R - \epsilon \sigma_R T^4} (\Delta' + \gamma_R) \quad (5.10)$$

The quantity $q_e^R - \epsilon \sigma_R T^4$ is the radiation flux incident on the wall after subtraction of the reflected radiation flux from the wall without allowing for its absorption in the boundary layer.

Formula (5.10) contains two unknown quantities T_w and γ_R . Their determination is a fundamental problem both in theory and experiment.

§ 6. PROBLEM ON A CHEMICALLY BALANCED FLOW IN A BOUNDARY LAYER OVER A NONDISINTEGRATING SURFACE HAVING IDEAL CATALYTIC CHARACTERISTICS

This problem can be reduced to jointly solving Equations (2.13), (2.14),

(6) In the presence of radiation the law of decrease in the convective and diffusion heat fluxes with flow has not yet been studied, but from the general arguments it follows that with a sufficiently large flow these fluxes can sometimes be ignored in comparison with the radiation fluxes q_w^R , reaching the wall.

(2.23), (2.30'), (2.46), (2.42') with the boundary conditions and initial conditions (2.57), (2.58), (2.63), (2.64), (2.65), (2.66), (2.67), (3.4).

In the case of a chemical balance in the flow for the conditions of balance (2.66), (2.67) we can consider the existence of N_r integrals of the basic system of Equations (2.13) - (2.22) of the boundary layer. These integrals can be used to eliminate from the energy equation (2.41) or (2.42) the N_r diffusion fluxes \dot{m}_i ($i = 1, \dots, N_r$) and N_r derivatives of the concentrations of $\frac{\partial c_i}{\partial y}$ ($i = 1, \dots, N_r$) reaction products. For this let us represent Conditions (2.66), (2.67) by the molar concentrations

$$\prod_{j=N_r+1}^N \frac{x_j^{v_{ij}}}{x_i} = \frac{K_{p,i}(T)}{p^{v_i}} \quad (i = 1, \dots, N_r) \quad (6.1) \quad /30$$

The Saha conditions (2.67) are a particular form of the Guldberg-Waage conditions (2.66). Therefore, (6.1) will include all Conditions (2.66) and (2.67). If we substitute the solution into Expression (6.1), they can be transformed into identities. They can therefore be differentiated. If we take the logarithm of (6.1) and then formulate the gradient and the isobaric equations of Van't Hoff⁽⁷⁾ for each of the reactions (2.1)

$$\frac{d \ln K_p}{dT} = \frac{Q_i^*}{R_A T^2} \quad [Q_i^*] = \frac{\text{cal}}{\text{mole}} \quad (i = 1, \dots, N_r) \quad (6.2)$$

we then find

$$\frac{1}{x_i} \nabla x_i - \sum_{j=N_r+1}^N \frac{v_{ij}}{x_j} \nabla x_j = - \frac{Q_i^*}{R_A T} \nabla \ln T + v_i \nabla \ln p \quad (6.3)$$

(7) For the ionization reactions the equations in (6.2) will differ only in the form of the right-hand side.

If we substitute ∇x_i here from (2.44') and assume that the charge in the reactions is conserved, i.e.,

$$e_l - \sum_{j=N_r+1}^N \nu_{lj} g_j \equiv 0 \quad (l=1, \dots, N_r)$$

Then

$$\sum_{j=1}^N x_j a_{ij} \left(\frac{\vec{J}_j}{\rho_j} - \frac{\vec{J}_i}{\rho_i} \right) - \sum_{j=N_r+1}^N \nu_{ij} \sum_{k=1}^N x_k a_{kj} \left(\frac{\vec{J}_k}{\rho_k} - \frac{\vec{J}_j}{\rho_j} \right) = - \frac{m_i}{RAT} Q_i' \nabla \ln T - \tilde{m}_i \nabla \ln p \quad (6.4)$$

where

$$\tilde{m}_i = \frac{m_i}{m} - \sum_{j=N_r+1}^N \nu_{ij} \frac{m_j}{m} \equiv 0 \quad (i=1, \dots, N)$$

Let us introduce from the definitions in (2.23) the diffusion fluxes of elements \vec{J}_j^* ($j=N_r+1, \dots, N$) instead of the diffusion fluxes of the base components \vec{J}_j ($j=N_r+1, \dots, N$).

$$\vec{J}_j = \vec{J}_j^* - \sum_{k=1}^{N_r} \frac{\nu_{kj} m_j}{m_k} \vec{J}_k \quad (j=N_r+1, \dots, N) \quad (6.5)$$

We make this substitution, since the fluxes \vec{J}_j^* ($j=N_r+1, \dots, N$) satisfy the homogeneous equation⁽⁸⁾ (2.23), and in the case of a nondisintegrating wall they satisfy the zero boundary conditions (2.65). Therefore, we can expect that they will differ little in this case from zero in the flux. When the mass (elements) comes from the wall, we must expect that \vec{J}_j^* ($j=N_r+1, \dots, N$) varies more smoothly in comparison with the fluxes \vec{J}_j ($j=N_r+1, \dots, N$).

(8) In this case our arguments are not confined to the framework of the approximate boundary layer theory but have a general character.

since the chemical reactions directly influence these latter. If we drop \vec{J}_j^* ($j=N+1, \dots, N$) from (6.4) with the aid of (6.5) we find the following linear algebraic system for the diffusion fluxes of the components of the reaction products

$$\begin{aligned} & \frac{\vec{J}_i}{m_i} \sum_{k=1}^N \frac{\alpha_k \Delta_{ik}}{\alpha_i} + \sum_{k=1}^{N_r} \frac{\vec{J}_k}{m_k} \left[\sum_{j=N_r+1}^N \sum_{e=N_r+1}^N \frac{\nu_{ij} \Delta_{je}}{\alpha_j} (\nu_{kj} \alpha_e - \nu_{ke} \alpha_j) + \right. \\ & + \sum_{j=N_r+1}^N \sum_{e=1}^{N_r} \nu_{ij} \nu_{kj} \frac{\alpha_e \Delta_{je}}{\alpha_j} + \sum_{j=N_r+1}^N (\nu_{kj} \Delta_{ij} + \nu_{ij} \Delta_{jk}) - \Delta_{ik} \left. \right] = \frac{m_i Q_i'}{R_A T} \nabla \ln T + \\ & + \sum_{j=N_r+1}^N \frac{\vec{J}_j^*}{m_j} \left(\Delta_{ij} + \sum_{k=1}^N \frac{\alpha_k \nu_{ij} \Delta_{jk}}{\alpha_j} - \sum_{e=N_r+1}^N \nu_{ie} \Delta_{je} \right), \\ & \Delta_{ij} = \frac{Q_{ij}}{n} \end{aligned}$$

This system can be written more briefly

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$$\sum_{k=1}^{N_r} A_{ik} \frac{\vec{J}_k}{m_k} = \frac{m_i Q_i'}{R_A T} \frac{\partial \ln T}{\partial y} + \sum_{j=N_r+1}^N \alpha_j \frac{\vec{J}_j^*}{m_j} \quad (i=1, \dots, N_r) \quad (6.6)$$

where

$$\begin{aligned} A_{ik} &= \sum_{j=N_r+1}^N \sum_{e=N_r+1}^N \frac{\nu_{ij} \Delta_{je}}{\alpha_j} (\nu_{kj} \alpha_e - \nu_{ke} \alpha_j) + \sum_{j=N_r+1}^N \sum_{e=1}^{N_r} \nu_{ij} \nu_{kj} \frac{\alpha_e \Delta_{je}}{\alpha_j} + \sum_{j=N_r+1}^N (\nu_{kj} \Delta_{ij} + \\ & + \Delta_{kj} \nu_{ij}) - \Delta_{ik}, \quad Q_i' = Q_i + \sigma_i^T \frac{R_A T}{m_i} \\ C_{ij} &= \Delta_{ij} + \sum_{k=1}^N \frac{\alpha_k \nu_{ij} \Delta_{jk}}{\alpha_j} - \sum_{e=N_r+1}^N \nu_{ie} \Delta_{je} \quad (i, k=1, \dots, N_r) \end{aligned} \quad (6.7)$$

Let us show that the matrix $\|A_{ik}\|$ ($i, k=1, \dots, N_r$) is symmetrical⁽⁹⁾. For this let us transform the first term in the expression A_{ik} .

(9) This proof agrees with that in Reference [16] where the effective coefficient of thermal conductivity was computed in a stationary chemically reacting mixture of gases.

$$\begin{aligned}
& \sum_{j=N_k+1}^N \sum_{e=N_k+1}^N \frac{v_{ij} \Delta_{je}}{x_j} (v_{kj} x_e - v_{ke} x_j) = \frac{1}{2} \left(\sum_j \sum_e + \sum_e \sum_j \right) = \\
& = \frac{1}{2} \sum_{j=N_k+1}^N \sum_{e=N_k+1}^N \left[\frac{v_{ij} \Delta_{je}}{x_j} (x_e v_{kj} - v_{ke} x_j) + \frac{v_{ie} \Delta_{je}}{x_e} (v_{ke} x_j - v_{kj} x_e) \right] = \\
& = \sum_{j=N_k+1}^N \sum_{e=N_k+1}^N \Delta_{je} \frac{(v_{kj} x_e - v_{ke} x_j)(v_{ij} x_e - v_{ie} x_j)}{x_j x_e} = \\
& = \frac{1}{2} \sum_{j=N_k+1}^{N-1} \sum_{e=j+1}^N \Delta_{je} \frac{(v_{kj} x_e - v_{ke} x_j)(v_{ij} x_e - v_{ie} x_j)}{x_j x_e}
\end{aligned}$$

Then

$$\begin{aligned}
A_{ik} &= \frac{1}{2} \sum_{j=N_k+1}^N \sum_{e=N_k+1}^N \Delta_{je} \frac{(v_{kj} x_e - v_{ke} x_j)(v_{ij} x_e - v_{ie} x_j)}{x_j x_e} + \\
&+ \sum_{j=N_k+1}^N \sum_{e=1}^{N_k} v_{ij} v_{kj} \frac{x_e}{x_j} \Delta_{je} + \sum_{j=N_k+1}^N (v_{kj} \Delta_{ij} + v_{ij} \Delta_{kj}) - \Delta_{ik}
\end{aligned}$$

Hence, it is immediately clear that

$$A_{ik} = A_{ki} \quad (i, k = 1, \dots, N_k) \quad (6.8)$$

We can find A_{ii}

$$\begin{aligned}
A_{ii} &= \sum_{k=1}^N \frac{x_k \Delta_{ik}}{x_i} + \frac{1}{2} \sum_{j=N_k+1}^N \sum_{e=N_k+1}^N \Delta_{je} \frac{(v_{ij} x_e - v_{ie} x_j)^2}{x_j x_e} + \sum_{j=N_k+1}^N \sum_{e=1}^{N_k} v_{ij}^2 \frac{x_e \Delta_{je}}{x_j} + \\
&+ 2 \sum_{j=N_k+1}^N v_{ij} \Delta_{ij} - \Delta_{ii} = \frac{1}{2} \sum_{j=N_k+1}^N \sum_{e=N_k+1}^N \Delta_{je} \frac{(v_{ij} x_e - v_{ie} x_j)^2}{x_j x_e} + \sum_{j=N_k+1}^N \sum_{\substack{e=1 \\ e \neq i}}^{N_k} v_{ij}^2 \frac{x_e \Delta_{je}}{x_j} + \\
&+ \sum_{j=N_k+1}^N \Delta_{ij} \frac{(x_j + v_{ij} x_i)^2}{x_i x_j} + \sum_{\substack{k=1 \\ k \neq i}}^{N_k} \frac{x_k \Delta_{ik}}{x_i}
\end{aligned} \quad (6.9)$$

Thus, there are altogether $1/2 N_k (N_k + 1)$ different coefficients rather than 32

N_r^2 . From the system of consistent linear equations (6.6) we can find N_r diffusion fluxes of the reaction products

$$\frac{\vec{J}_i}{m_i} = \frac{\begin{vmatrix} A_{11} & A_{12} & \dots & A_{1i-1} & m_i Q_i' & A_{1i+1} & \dots & A_{1N_r} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ A_{N_r1} & A_{N_r2} & \dots & A_{N_ri-1} & m_{N_r} Q_{N_r}' & A_{N_ri+1} & \dots & A_{N_r N_r} \end{vmatrix}}{\text{Det } \|A_{ij}\|} \frac{\nabla \ln T}{RAT} +$$

$$+ \frac{\begin{vmatrix} A_{11} & A_{12} & \dots & A_{1i-1} & \sum_{j=N_r+1}^N c_{1j} \frac{\vec{J}_j^*}{m_j} & A_{1i+1} & \dots & A_{1N_r} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ A_{N_r1} & A_{N_r2} & \dots & A_{N_ri-1} & \sum_{j=N_r+1}^N c_{N_rj} \frac{\vec{J}_j^*}{m_j} & A_{N_ri+1} & \dots & A_{N_r N_r} \end{vmatrix}}{\text{Det } \|A_{ij}\|} \quad (i=1, \dots, N_r) \quad (6.10)$$

The remaining fluxes are found from the equations in (6.5), the solution of which gives

$$\frac{\vec{J}_j}{m_j} = \frac{\begin{vmatrix} 0 & v_{1j} & \dots & v_{N_rj} \\ m_i Q_i' & A_{11} & \dots & A_{1N_r} \\ \vdots & \vdots & & \vdots \\ m_{N_r} Q_{N_r}' & A_{N_r1} & \dots & A_{N_r N_r} \end{vmatrix}}{\text{Det } \|A_{ij}\|} \frac{\nabla \ln T}{RAT} +$$

$$+ \frac{\begin{vmatrix} 0 & v_{1j} & \dots & v_{N_rj} \\ \sum_{j=N_r+1}^N c_{1j} \frac{\vec{J}_j^*}{m_j} & A_{11} & \dots & A_{1N_r} \\ \vdots & \vdots & & \vdots \\ \sum_{j=N_r+1}^N c_{N_rj} \frac{\vec{J}_j^*}{m_j} & A_{N_r1} & \dots & A_{N_r N_r} \end{vmatrix}}{\text{Det } \|A_{ij}\|} +$$

$$+ \frac{y_j^*}{m_j} \quad (j = N_r+1, \dots, N) \quad (6.11)$$

This heat flux along the y-axis will then be $(\frac{\partial p}{\partial y} = 0)$

$$\begin{aligned}
 q = & -\lambda \frac{\partial T}{\partial y} + R_A T \sum_{j=1}^N \sum_{k=1}^N \frac{\alpha_j \partial_k^T \alpha_{kj}}{m_k} (V_k - V_j) = -\lambda \frac{\partial T}{\partial y} - R_A T \sum_{k=1}^{N_c} \bar{G}_k^T \frac{\gamma_k}{m_k} - \\
 & - R_A T \sum_{j=N_c+1}^N \frac{\gamma_j^* \sum_{i=1}^N \gamma_i^T}{m_j} = \\
 & = -\lambda \frac{\partial T}{\partial y} + \frac{\begin{vmatrix} 0 & \bar{G}_1^T & \dots & \bar{G}_{N_c}^T \\ m_1 Q_1' & A_{11} & \dots & A_{1N_c} \\ \dots & \dots & \dots & \dots \\ m_{N_c} Q_{N_c}' & A_{N_c 1} & \dots & A_{N_c N_c} \end{vmatrix}}{\text{Det } \|A_{ij}\|} \frac{\partial \ln T}{\partial y} + \frac{\begin{vmatrix} 0 & \bar{G}_1^T & \dots & \bar{G}_{N_c}^T \\ \sum_{j=N_c+1}^N c_{ij} \frac{\gamma_j^*}{m_j} & A_{11} & \dots & A_{1N_c} \\ \dots & \dots & \dots & \dots \\ \sum_{j=N_c+1}^N c_{N_c j} \frac{\gamma_j^*}{m_j} & A_{N_c 1} & \dots & A_{N_c N_c} \end{vmatrix}}{\text{Det } \|A_{ij}\|} R_A T - \quad (6.12)
 \end{aligned}$$

If we ignore the diffusion thermoeffect, then only the first single term remains in (6.12). The group of terms standing under the y derivative in Equation (2.41) is written by allowing for (6.10) in the form

$$\lambda \frac{\partial T}{\partial y} + \sum_{k=1}^{N_c} Q_k' \gamma_k + R_A T \sum_{j=N_c+1}^N \frac{\gamma_j^*}{m_j} \sum_j^T = \lambda_{\text{eff}} \frac{\partial T}{\partial y} + \sum_{j=N_c+1}^N \left(\square_j + R_A T \sum_j^T \right) \frac{\gamma_j^*}{m_j} \quad (6.13)$$

where

$$\begin{aligned}
 \lambda_{\text{eff}} &= \lambda + \lambda_r \\
 \lambda &= \lambda' + R_A \sum_{k=1}^N \frac{\partial_k^T}{m_k} \sum_k^T
 \end{aligned}$$

(6.14) /33

$$\begin{aligned}
 \lambda_r &= \frac{1}{R_A T^2} \frac{\begin{vmatrix} 0 & m_1 Q_1' & \dots & m_{N_c} Q_{N_c}' \\ m_1 Q_1' & A_{11} & \dots & A_{1N_c} \\ \dots & \dots & \dots & \dots \\ m_{N_c} Q_{N_c}' & A_{N_c 1} & \dots & A_{N_c N_c} \end{vmatrix}}{\text{Det } \|A_{ij}\|}, \quad \square_j = - \frac{\begin{vmatrix} 0 & m_1 Q_1' & \dots & m_{N_c} Q_{N_c}' \\ c_{1j} & A_{11} & \dots & A_{1N_c} \\ \dots & \dots & \dots & \dots \\ c_{N_c j} & A_{N_c 1} & \dots & A_{N_c N_c} \end{vmatrix}}{\text{Det } \|A_{ij}\|} \quad (6.15)
 \end{aligned}$$

The coefficient λ is expressed through the expansion coefficients in the Sonin polynomial and is finally expressed in the form of ratio of determinants, whose elements are integrals of the paired collisions [8]. Even if we ignore the diffusion thermoeffect, the contribution of the diffusion fluxes of the elements to the total heat flux may be significant. Even when $\gamma_j^* = 0$ ($j = N_c + 1, \dots, N$), which will be the case in a stationary chemi-

cally reacting mixture of gases or in a mixture with homogeneous coefficients of diffusion with no transport of the material from the boundaries, Expression (6.13) will generalize to the known expression of the total heat flux given in Reference [4] for the case of allowing for the diffusion thermoeffect. If we use Expression (6.13) the right-hand side of the energy equation (2.41) can be represented only through the temperature and diffusion fluxes of the elements.

We can prove the following argument. If the coefficients of binary diffusion are identical for all components and the problem is solved under boundary conditions (2.65), then

$$\mathcal{Y}_j^* \equiv 0 \quad (j = N_k + 1, \dots, N) \quad (6.16)$$

The total heat flux in the direction of the y-axis can be represented in the form

$$\begin{aligned} \mathcal{Y}q &= q + \sum_{k=1}^N h_k \mathcal{Y}_k = q - \sum_{k=1}^{N_k} Q_k \mathcal{Y}_k + \sum_{j=N_k+1}^N h_j \mathcal{Y}_j^* = \\ &= -\lambda \frac{\partial T}{\partial y} - \sum_{k=1}^{N_k} Q_k' \mathcal{Y}_k + \sum_{j=N_k+1}^N \left(h_j - \frac{R_A T}{m_j} \sum_j^T \right) \mathcal{Y}_j^* \end{aligned} \quad (6.17)$$

Then from the preceding argument and (6.17) it follows that if the coefficients of binary diffusion are identical for all components and the problem is solved for boundary conditions (2.65), then the expression for the convective heat flux in the direction of the y-axis in the boundary layer agrees with the respective expression in the stationary chemically reacting mixture of gases.

In order to finally write Equation (2.41) in terms of the temperature for a chemically balanced boundary layer, it is necessary that the derivatives of the function H be expressed in terms of the respective derivatives of the temperature. We have

$$dH = q_p dT - \sum_{k=1}^{N_r} Q_k dc_k + \sum_{j=N_r+1}^N (y_j^* - c_j^*) c_{pj} dT \quad (6.18)$$

From this expression it is clear that in turn it is necessary to express the partial derivatives of the concentrations C_i ($i = 1, \dots, N_r$) of the reaction products in terms of the temperature derivatives. If we drop the diffusion fluxes \vec{J}_j ($j = N_r + 1, \dots, N$) from the Stefan - Maxwell Expressions (2.44) and (2.46), with the aid of (6.5)

$$\frac{1}{\alpha_i} \nabla \alpha_i = - \frac{\vec{J}_i}{m_i} \sum_{j=1}^N \frac{\alpha_j \Delta_{ij}}{\alpha_i} + \sum_{e=1}^{N_r} \tilde{\Delta}_{ie}^{**} \frac{\vec{J}_e}{m_e} + \sum_{j=N_r+1}^N \Delta_{ij}^{**} \frac{\vec{J}_j^*}{m_j} + \nabla \ln T \sum_i^{**} + \nabla \ln p \pi_i^{**} \quad (6.19)$$

where

$$\begin{aligned} \Delta_{ij} &= \frac{\alpha_{ij}}{n}, \quad \Delta_{ij}^* = \Delta_{ij} - e_i^* \sum_{k=1}^N (e_k - e_j) x_k \Delta_{kj} \\ \tilde{\Delta}_{ie}^{**} &= \Delta_{ie}^{**} - \sum_{j=N_r+1}^N v_{ej} \Delta_{ij}^{**}, \quad \sum_i^{**} = \sum_i^T - e_i^* \sum_{k=1}^N x_k e_k \sum_k^T \\ \pi_i^{**} &= \frac{m_i}{m} - 1 - e_i^* \sum_{k=1}^N c_k e_k \quad (i, j = 1, \dots, N) \end{aligned} \quad (6.20)$$

$$\frac{1}{\alpha} \nabla \alpha_i = - \frac{\vec{J}_i}{m_i} \sum_{j=1}^N \frac{\alpha_j}{\alpha_i} \Delta_{ij} + \sum_{e=1}^{N_r} \tilde{\Delta}_{ie}^* \frac{\vec{J}_e}{m_e} + \sum_{j=N_r+1}^N \Delta_{ij}^* \frac{\vec{J}_j^*}{m_j} + \nabla \ln T \sum_i^* + \nabla \ln p \pi_i^* \quad (6.21) \quad /34$$

where

$$\begin{aligned} \Delta_{ij}^* &= \Delta_{ij} - \sum_{k=1}^N x_k \Delta_{kj} \left(\frac{m_k}{m} - \frac{m_j}{m} \right) - (e_i^* - \gamma) \sum_{k=1}^N x_k \Delta_{kj} (e_k - e_j) \\ \tilde{\Delta}_{ie}^* &= \Delta_{ie}^* - \sum_{j=N_r+1}^N v_{ej} \Delta_{ij}^*, \quad \sum_i^* = \sum_i^T - \sum_{k=1}^N c_k \sum_k^T - (e_i^* - \gamma) \sum_{k=1}^N x_k e_k \sum_k^T \\ \pi_i^* &= \frac{m_i}{m} - \sum_{k=1}^N c_k \frac{m_k}{m} - (e_i^* - \gamma) \sum_{k=1}^N c_k e_k \end{aligned} \quad (6.22)$$

For now let us compute the intermediate values using (6.10)

$$\sum_{e=1}^{N_e} \tilde{\Delta}_{ie}^* \frac{\tilde{J}_e}{m_e} = - \square_i (m Q') \nabla \ln T - \sum_{j=N_e+1}^N \square_j (c_j) \frac{\tilde{J}_j^*}{m_j} \quad (6.23)$$

where

$$\square_i (m Q') = \frac{\begin{vmatrix} 0 & \tilde{\Delta}_{i1}^* & \dots & \tilde{\Delta}_{iN_e}^* \\ m_1 Q_1' & A_{11} & \dots & A_{1N_e} \\ \dots & \dots & \dots & \dots \\ m_{N_e} Q_{N_e}' & A_{N_e 1} & \dots & A_{N_e N_e} \end{vmatrix}}{\text{Det } \| A_{ij} \|} \frac{1}{R_A T}, \quad \square_j (c_j) = \frac{\begin{vmatrix} 0 & \tilde{\Delta}_{11}^* & \dots & \tilde{\Delta}_{1N_e}^* \\ A_{11} & \dots & A_{1N_e} \\ \dots & \dots & \dots \\ A_{N_e 1} & \dots & A_{N_e N_e} \end{vmatrix}}{\text{Det } \| A_{ij} \|} \quad (6.24)$$

where

$$\sum_{i=1}^{N_e} \frac{\tilde{J}_i}{m_i} Q_i \sum_{j=1}^N \frac{m_j}{m_j} c_j \Delta_{ij} = - \square (m Q', Q) \nabla \ln T - \sum_{j=N_e+1}^N \square_j (c_j) \frac{\tilde{J}_j^*}{m_j}$$

(6.25)

$$\square (m Q', Q) = \frac{\begin{vmatrix} 0 & Q_1 \sum_{j=1}^N \frac{m_j}{m_j} c_j \Delta_{1j} & \dots & Q_{N_e} \sum_{j=1}^N \frac{m_{N_e j}}{m_j} \Delta_{N_e j} c_j \\ m_1 Q_1' & A_{11} & \dots & A_{1N_e} \\ \dots & \dots & \dots & \dots \\ m_{N_e} Q_{N_e}' & A_{N_e 1} & \dots & A_{N_e N_e} \end{vmatrix}}{\text{Det } \| A_{ij} \|} \quad (6.26)$$

$$\square(c, Q) = \frac{\begin{vmatrix} 0 & Q_1 \sum_{j=1}^N \frac{m_1}{m_j} c_j \Delta_{1j} & \dots & Q_{N_r} \sum_{j=1}^{N_r} \frac{m_{N_r}}{m_j} c_j \Delta_{N_r j} \\ c_{1j} & A_{11} & \dots & A_{1 N_r} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N_r j} & A_{N_r 1} & \dots & A_{N_r N_r} \end{vmatrix}}{\text{Det } \|A_{ij}\|}$$

Using (6.21), (6.23) and (6.25) we obtain

$$\begin{aligned} \sum_{k=1}^{N_r} Q_k \nabla Q_k &= \square(mQ, Q) \nabla \ln T + \sum_{j=N_r+1}^N \square_j(c, Q) \frac{\vec{J}_j^*}{m_j} - \\ &- \sum_{i=1}^{N_r} \alpha_i \square_i(mQ) \nabla \ln T - \sum_{i=1}^{N_r} \sum_{j=N_r+1}^N \alpha_i \square_j(c_{ij}) \frac{\vec{J}_j^*}{m_j} + \\ &+ \sum_{i=1}^{N_r} \sum_{j=N_r+1}^N \alpha_i \Delta_{ij} \frac{\vec{J}_j^*}{m_j} + \nabla \ln T \sum_{i=1}^{N_r} \alpha_i \sum_i^T + \nabla \ln p \sum_{i=1}^{N_r} \alpha_i \pi_i^* = \\ &= A \nabla T + \sum_{j=N_r+1}^N B_j \vec{J}_j^* + \mathcal{D} \nabla p \end{aligned} \quad (6.27)$$

where

$$\begin{aligned} A &= \square(mQ, Q) - \sum_{i=1}^{N_r} \alpha_i \square_i(mQ) + \sum_{i=1}^{N_r} \alpha_i \sum_i^T \\ B_j m_j &= \square_j(c, Q) - \sum_{i=1}^{N_r} \alpha_i \square_j(c_{ij}) + \sum_{i=1}^{N_r} \alpha_i \Delta_{ij} \\ \mathcal{D} p &= \sum_{i=1}^{N_r} \alpha_i \pi_i^* \end{aligned} \quad (6.28)$$

We have expressed ∇C_i ($i = 1, \dots, N_r$) in terms of ∇T , ∇p and \vec{J}_j^* ($j = 1, \dots, N_r$). However, we can find the equivalent expression for ∇C_i ($i = 1, \dots, N_r$) in terms of ∇T , ∇p and C_j^* . In fact with the specified field of element concentrations C_j^* ($j = N_r + 1, \dots, N$) the composition of

the gas must be found from the solution to the transcendental system of equations

$$\begin{aligned} \bar{c}_j + \sum_{k=1}^{N_r} \nu_{kj} \bar{c}_k &= \bar{c}_j^* \quad (j = N_r+1, \dots, N) \\ \prod_{j=N_r+1}^N c_j^{\nu_{ij}} c_i^{-1} &= K_{p,i}(T) P^{-\nu_i} m^{-\nu_i} \quad (i = 1, \dots, N_r) \end{aligned} \quad (6.29)$$

where

$$\nu_i = \sum_{j=N_r+1}^N \nu_{ij} - 1, \quad \bar{c}_i = \frac{c_i}{m_i} \quad (i = 1, \dots, N), \quad c_j^* = \frac{c_j^*}{m_j} \quad (j = N_r+1, \dots, N) \quad (6.30)$$

If we take the logarithm of the second system (6.29) and formulate the gradient, we find

$$\frac{1}{\bar{c}_i} \nabla \bar{c}_i - \sum_{j=N_r+1}^N \nu_{ij} \frac{\nabla \bar{c}_j}{\bar{c}_j} = -\frac{Q_i^*}{RAT} \nabla \ln T + \nu_i \nabla \ln p - \nu_i m \sum_{k=1}^N \nabla c_k \quad (6.31)$$

If we eliminate $\nabla \bar{c}_j$ ($i = 1, \dots, N$) from these equations with the aid of the first system (6.29), we find the consistent system for determining ∇c_i ($i = 1, \dots, N_r$)

$$\sum_{k=1}^{N_r} B_{ik} \nabla \bar{c}_k = \sum_{j=N_r+1}^N \left(\frac{\nu_{ij}}{\bar{c}_j} - \nu_i m \right) \nabla c_j^* - \frac{Q_i^*}{RAT} \nabla \ln T + \nu_i \nabla \ln p \quad (i = 1, \dots, N_r) \quad (6.32)$$

where

$$\begin{aligned} B_{ik} = B_{ki} &= \sum_{j=N_r+1}^N \nu_{kj} \left(\frac{\nu_{ij}}{\bar{c}_j} - \nu_i m \right) = \sum_{j=N_r+1}^N \frac{\nu_{ij} \nu_{kj}}{\bar{c}_j} - \nu_i \nu_k m \\ B_{ii} &= \frac{1}{\bar{c}_i} + \sum_{j=N_r+1}^N \frac{\nu_{ij}^2}{\bar{c}_j} - \nu_i^2 m \end{aligned} \quad (6.33) \quad \frac{136}{69}$$

From the system of consistent linear equations (6.32) we find N_r gradients $\nabla \bar{G}_i$ ($i = 1, \dots, N_r$) in the form

$$\nabla \bar{G}_i = \frac{\begin{vmatrix} B_{11} \dots B_{1i-1} \sum_{j=N_r+1}^N \bar{v}_{ij} \nabla \bar{G}_j^* & B_{1i+1} \dots B_{1N_r} \\ \dots & \dots \\ B_{N_r 1} & B_{N_r i-1} \sum_{j=N_r+1}^N \bar{v}_{N_r j} \nabla \bar{G}_j^* & B_{N_r i+1} \dots B_{N_r N_r} \end{vmatrix}}{\text{Det } \|A_{ij}\|} \quad (6.34)$$

$$+ \nabla \ln p \left[\frac{\begin{vmatrix} B_{11} \dots B_{1i-1} v_1 & B_{1i+1} \dots B_{1N_r} \\ \dots & \dots \\ B_{N_r 1} \dots B_{N_r i-1} v_{N_r} & B_{N_r i+1} \dots B_{N_r N_r} \end{vmatrix}}{\text{Det } \|A_{ij}\|} - \frac{\begin{vmatrix} B_{11} \dots B_{1i-1} m_1 Q_1 & B_{1i+1} \dots B_{1N_r} \\ \dots & \dots \\ B_{N_r 1} \dots B_{N_r i-1} m_{N_r} Q_{N_r} & B_{N_r i+1} \dots B_{N_r N_r} \end{vmatrix}}{\text{Det } \|A_{ij}\|} \right] \frac{\nabla \ln T}{R_A T}$$

$$\bar{v}_{ij} = \frac{v_{ij}}{c_j} - v_i \cdot m \quad (i=1, \dots, N_r).$$

The gradients of the remaining concentrations can be found from the first system of equations (6.29)

$$\nabla \bar{G} = \nabla \bar{G}^* + \frac{\begin{vmatrix} 0 & v_{1j} \dots v_{N_r j} \\ \sum_{j=N_r+1}^N \bar{v}_{ij} \nabla \bar{G}_j^* & B_{11} \dots B_{1N_r} \\ \dots & \dots \\ \sum_{j=N_r+1}^N \bar{v}_{N_r j} \nabla \bar{G}_j^* & B_{N_r 1} \dots B_{N_r N_r} \end{vmatrix}}{\text{Det } \|B_{ij}\|} + \frac{\begin{vmatrix} 0 & v_{1j} \dots v_{N_r j} \\ m_1 Q_1 & B_{11} \dots B_{1N_r} \\ \dots & \dots \\ m_{N_r} Q_{N_r} & B_{N_r 1} \dots B_{N_r N_r} \end{vmatrix}}{\text{Det } \|B_{ij}\|} \frac{\nabla \ln T}{R_A T}$$

$$+ - \frac{\begin{vmatrix} 0 & v_{1j} \dots v_{N_r j} \\ v_1 & B_{11} \dots B_{1N_r} \\ \dots & \dots \\ v_{N_r} & B_{N_r 1} \dots B_{N_r N_r} \end{vmatrix}}{\text{Det } \|B_{ij}\|} \nabla \ln p \quad (6.35)$$

Formulas (6.34) and (6.35) differ from the respective formulas of the theory of thermodynamic computation of a chemically reacting mixture of gases for specified P and T , in that here we take into account the change in the elementary chemical composition and its influence on the composition of the gas,

i.e., here we look at the so-called open thermodynamic system, when along with the P and T determining parameters there are still the N_e parameters

$$C_{N+1}^*, \dots, C_N^*$$

Using the Solution (6.34), we find

$$-\sum_{k=1}^{N_e} Q_k \frac{\partial c_k}{\partial y} = \sum_{j=N_e+1}^N \square_j (\bar{v}_{ij} Q_i) \frac{\partial q_j^*}{\partial y} - \square(mQ, Q) \frac{\partial T}{\partial y} \quad (6.36)$$

where

$$\square_j(\bar{v}_{ij}, Q) = \frac{\begin{vmatrix} 0 & Q_1 & \dots & Q_{N_e} \\ \bar{v}_{ij} & B_{11} & \dots & B_{1N_e} \\ \dots & \dots & \dots & \dots \\ \bar{v}_{N_e j} & B_{N_e 1} & \dots & B_{N_e N_e} \end{vmatrix}}{\text{Det } \|B_{ij}\|}, \quad \square(mQ, Q) = \frac{\begin{vmatrix} 0 & Q_1 & \dots & Q_{N_e} \\ m_1 Q_1 & B_{11} & \dots & B_{1N_e} \\ \dots & \dots & \dots & \dots \\ m_{N_e} Q_{N_e} & B_{N_e 1} & \dots & B_{N_e N_e} \end{vmatrix}}{\text{Det } \|B_{ij}\|} \quad (6.37) \quad 137$$

Then from (6.18), we find

$$C_p^{\text{eff}} = C_p - \square(mQ, Q) + \sum_{j=N_e+1}^N (c_j^* - q_j^*) c_{pj}$$

where

$$\frac{\partial H}{\partial y} = C_p^{\text{eff}} \frac{\partial T}{\partial y} + \sum_{j=N_e+1}^N \square_j(\bar{v}_{ij}, Q) \frac{\partial q_j^*}{\partial y}$$

Using (6.14) and (6.38), the energy equation (2.41) for a chemically balanced boundary layer may be written in terms of the temperature. However, for convenience in the future, we retain the convective operator in Equation (2.41) without change, but convert the right-hand side

$$\rho u \frac{\partial}{\partial x} (H - H_e) + \rho v \frac{\partial}{\partial y} (H - H_e) + \sum_{j=N_e+1}^N \left\{ \rho u \frac{\partial}{\partial x} [(c_j^* - q_j^*) h_j] + \rho v \frac{\partial}{\partial y} [(c_j^* - q_j^*) h_j] + \frac{\partial}{\partial y} (q_j^* h_j) \right\} = \frac{\partial}{\partial y} \left[(\lambda + \lambda_e) \frac{\partial T}{\partial y} + \mu \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) + \sum_{j=N_e+1}^N (R_A T \Sigma_j^T + \square_j) \frac{q_j^*}{m_j} \right] \quad (6.40)$$

The energy equation (2.42), written in terms of the enthalpy, will be

$$\begin{aligned} & \rho u \frac{\partial}{\partial x} (H - H_e) + \rho v \frac{\partial}{\partial y} (H - H_e) + \sum_{j=N_k+1}^N \left\{ \rho u \frac{\partial}{\partial x} [(g_j - g_e^*) h_j] + \rho v \frac{\partial}{\partial y} [(g_j^* - g_e^*) h_j] + \frac{\partial}{\partial y} (J_j^* h_j) \right\} \\ &= \frac{\partial}{\partial y} \left\{ \frac{\lambda}{c_p} \left[\frac{\partial}{\partial y} (H - H_e) + (G-1) \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) - \sum_{j=N_k+1}^N (g_j^* - g^*) c_{pj} \frac{\partial T}{\partial y} \right] + \right. \\ & \quad \left. + \left(\lambda_k + \frac{\lambda}{c_p} A \right) \frac{\partial T}{\partial y} + \sum_{j=N_k+1}^N \left(R_A T \Sigma_j^T + \square_j + B_j \frac{m \lambda}{c_p} \right) \frac{J_j^*}{m_j} \right\} \end{aligned} \quad (6.41)$$

where the temperature gradient is related to the enthalpy gradient by Expression (6.38).

After solving the above problem, the friction is found from Expression (3.5). The total convective heat flux through the wall, according to (6.17), (6.15), (2.65), (6.32) will be equal to (the index "w" is omitted)

$$(-J_q)_w = \lambda \frac{\partial T}{\partial y} + \sum_{k=1}^{N_k} Q_k' J_k = (\lambda + \lambda_k) \frac{\partial T}{\partial y} = \frac{\lambda + \lambda_k}{c_p} \frac{\partial H}{\partial y} = \frac{\lambda + \lambda_k}{c_p} \Delta H \frac{\partial g}{\partial y} \quad (6.42)$$

where g is the relative total enthalpy

$$g = \frac{H - H_w}{H_e - H_w}$$

The transport coefficient of the total enthalpy (coefficient of heat transfer) then will be

$$C_H = \frac{(-J_q)_w}{\rho_e u_e \Delta H} = \frac{(\lambda + \lambda_k)_w}{c_p^{ef} \rho_e u_e} \left(\frac{\partial g}{\partial y} \right)_w$$

§ 7. PROBLEM ON A CHEMICALLY BALANCED FLOW IN A BOUNDARY LAYER OVER A DISINTEGRATING SURFACE POSSESSING AN IDEAL CATALYTIC CHARACTER

In this case the basic system of equations will be the same as in the preceding section. The boundary conditions at the outermost boundary of the boundary layer will be also retained as before. On the wall the conditions of attachment, Conditions (2.74), (2.66), (2.67) and Condition (4.2), must hold. In the case of balanced conditions on the wall, the diffusion fluxes on it may be taken from the expressions in (6.10). Then Condition (4.2) is rewritten as /38

$$-(\lambda + \lambda_w) \frac{\partial T}{\partial y} - q_e + 8G^* T^4 + \rho v (\Delta' + \gamma_R + \sum_{j=N+1}^N \frac{\square_j}{m_j} \Delta g_j^*) = 0 \quad (7.1)$$

or

$$-\frac{\lambda + \lambda_w}{\tilde{c}_p} \frac{\partial H}{\partial y} - q_e + 8G^* T^4 + \rho v (\Delta' + \gamma_R + \sum_{j=N+1}^N \frac{\square_j}{m_j} \Delta g_j^*) = 0 \quad (7.2)$$

In the process of reaching a solution, Condition (7.1) or (7.2) must participate as one of the boundary conditions. However, if it is possible to represent the coefficient of heat transport in the form

$$C_H = C_H^0 - \frac{(\rho v)_w}{\rho_e u_e} \gamma_H \quad (7.3)$$

where C_H^0 is the coefficient of heat transport in the absence of a blast, then

$$(\rho v)_w = \rho_e u_e C_H^0 \Delta H \frac{1 + \frac{q_e - 8G^* T^4}{\rho_e u_e C_H^0 \Delta H}}{\Delta'' + \gamma_R}, \quad \Delta'' = \Delta' + \sum_{j=N+1}^N \frac{\square_j}{m_j} \Delta g_j^* \quad (7.4)$$

The effective enthalpy of disintegration in a chemically balanced flow in the boundary layer and chemically balanced reactions on the wall will be

$$H_{\text{eff}} = \frac{\Delta'' + Y_R}{1 + \frac{q_e - \epsilon G^R T^4}{(-T_q)_w}} \quad (7.5)$$

It is interesting to peruse and follow the difference in the values of H_{eff} given by Formulas (5.3) and (7.5), which were obtained with only one difference: Formula (5.3) is for a chemically frozen flow inside the boundary layer and Formula (7.5) is for a balanced flow.

§ 8. DERIVATION OF THE SYSTEM OF INTEGRO-DIFFERENTIAL EQUATIONS EQUIVALENT TO THE ORIGINAL SYSTEM OF BOUNDARY LAYER DIFFERENTIAL EQUATIONS

In this section we shall derive a system of integro-differential equations of parabolic type, equivalent to the original system of boundary layer equations (2.13) - (2.22). On the basis of this system in § 10 we shall develop a method of successive approximations to obtain precise solutions to all the problems formulated in the preceding sections. On the basis of this method, we can also obtain approximate solutions (the first two approximations may be computed analytically).

Let us introduce the flow function $\psi(x, y)$ in the usual manner

$$\rho u' z^k = \frac{\partial \psi'}{\partial y'}, \quad -\rho v' z^k = \frac{\partial \psi'}{\partial x'} \quad (8.1)$$

where the primed values denote the redesignated variables which appear in the System (2.13) - (2.22).

The equation of continuity (2.13) is satisfied identically, and from

(2.14) we obtain a certain differential equation for the function ψ . Let us transform the momentum equation of thus obtained (2.14) for the flow function $\psi(x', y')$ using new independent variables

$$x' = x, \quad n = \frac{\eta}{\delta(x)}, \quad \eta = Ue x^k \int_0^y \rho(x, y') dy' \quad (8.2)$$

and a new unknown function $f(x, n)$

$$\psi'(x, y) = \delta(x) f(x, n) \quad (8.3)$$

where $\delta(x)$ is still an arbitrary function. Then in these variables Equation (2.14) will be

$$\delta^2 u \frac{\partial u}{\partial x} - (\delta' \delta f + \delta^2 \frac{\partial f}{\partial x}) \frac{\partial u}{\partial n} = \delta^2 \frac{d \ln Ue}{dx} \left(\frac{\rho_e}{\rho} - u^2 \right) + Ue x^{2k} \frac{\partial}{\partial y} \left(\mu \rho \frac{\partial u}{\partial y} \right) \quad (8.4)$$

where

$$u = \frac{\partial f}{\partial n}, \quad u = \frac{u'}{Ue}, \quad -\rho v' x^k = \delta f + \delta f_x + \delta u n_x \quad (8.5)$$

It is natural to also write Equation (8.4) in terms of the variables

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$$S = \int_0^x \mu_w \rho_w Ue x^{2k} dx, \quad n = n \quad (8.6)$$

in which it assumes a simpler form

$$\delta^2 u \frac{\partial u}{\partial S} - (\delta \delta'_S f + \delta^2 f'_S) \frac{\partial u}{\partial n} = \delta^2(S) \frac{d \ln Ue}{dS} \left(\frac{\rho_e}{\rho} - u^2 \right) + \frac{\partial}{\partial n} \left(l \frac{\partial u}{\partial n} \right), \quad l = \frac{\mu \rho}{\mu_w \rho_w} \quad (8.7)$$

If we set $\delta(S) = \sqrt{2S}$, then Equation (8.7) will be written in the Dorodnit-

syn - Stefanov variables in the Lees form [15].

Now let us integrate Equation (8.7) with respect to the coordinate n , from n to ∞ , and use the asymptotic function $u(s, n)$ and $f(s, n)$ at infinity [5] $\frac{\partial u}{\partial n} \rightarrow 0$, $f(s, n) \rightarrow n - \psi(s)$, $(1 - u)f \rightarrow 0$ when $n \rightarrow \infty$, then we find the following integro-differential equation

$$l \frac{\partial u}{\partial n} = \delta \delta_s' [(1-u)\varphi + \Delta(u)] + \delta^2 [(1-u) \frac{\partial \varphi}{\partial s} + \frac{\partial}{\partial s} \Delta(u)] + \delta^2 \beta \Delta(s) + \delta(1-u)\psi_w' \quad (8.8)$$

where

$$\varphi(s, n) = \int_n^\infty u(s, n') dn', \quad \psi_w' - (\delta f)'|_{n=0} = -(\delta v' z^k) \frac{dx}{ds} = -\frac{1}{\mu_w z^k} \cdot \frac{V_w}{u_e} \quad (8.9)$$

$$\Delta(u) = \int_n^\infty u(1-u) dn', \quad \Delta(s) = \int_n^\infty \left(\frac{\rho_e}{s} - u^2 \right) dn', \quad \beta = \frac{d \ln u_e}{ds} \quad (8.10)$$

The equations of the component diffusion (2.15) can each be transformed respectively to the independent variables

$$x' = x, \quad n_i' = \frac{\eta}{\delta_i'(\infty)} = \frac{\delta}{\delta_i'} n \quad (i=1, \dots, N) \quad (8.11)$$

and to the unknown functions (the primed values denote again the redesignated values which appear in (2.15))

$$\begin{aligned}
c_i'(x, y) &= c_i(x, n_i) & \gamma_i'(x, y) &= \frac{u_e r^k \rho_w \mu_w}{\delta_i(x)} \gamma_i \\
\psi'(x, y) &= \delta_i(x) f(x, n) = \delta_i(x) f_i(x, n_i) & f_i(x, n_i) &= \varphi_i(x, n_i) + f_{iw} \\
\varphi_i(x, n_i) &= \int_0^{n_i} u_i(x, n_i') dn_i' & u(x, n) &= u_i(x, n_i), \quad \delta f_w = \delta_i f_{iw} \\
u'(x, y) &= u_e u_i(x, n_i) = u_e u(x, n) & -g v r^k &= (\delta_i f_i)'_{\infty} + \delta_i u_i n_{i\infty}
\end{aligned} \tag{8.12}$$

where $\delta_1(x), \delta_2(x), \dots, \delta_N(x)$ are arbitrary functions. Then the equations in (2.15) along with Expressions (2.44) and (2.46) assume the following form

$$\delta_i^2 u_i \frac{\partial c_i}{\partial x} - (\delta_i \delta_i' f_i + \delta_i^2 \frac{\partial f_i}{\partial x}) \frac{\partial c_i}{\partial n_i} + \mu_w \rho_w u_e r^k \frac{\partial \gamma_i}{\partial n_i} = \frac{\dot{w}_i}{\rho u_e} \delta_i^2 \tag{8.13}$$

$$\begin{aligned}
\frac{e}{x_i} \frac{\partial x_i}{\partial n_i} &= - \sum_{j=1}^N x_j S_{ij} \frac{\gamma_j}{c_i} + \sum_{j=1}^N x_j d_{ij}^* \frac{\gamma_j}{c_j} \frac{\delta_i}{\delta_j} + d_i^* l \frac{\partial \ln T}{\partial n_i}, \quad S_{ij} = \frac{\mu}{\rho} a_{ij} \\
d_{ij}^* &= S_{ij} - e_i^* \sum_{k=1}^N x_k S_{kj} (e_k - e_j) \\
d_i^* &= \sum_{l=1}^T - e_i^* \sum_{k=1}^N x_k e_k \sum_k^T \\
&(i, j = 1, \dots, N)
\end{aligned} \tag{8.14}$$

$$\begin{aligned}
\frac{1}{c_i} \frac{\partial c_i}{\partial n_i} &= - \sum_{j=1}^N x_j S_{ij} \frac{\gamma_j}{c_i} + \sum_{j=1}^N x_j d_{ij} \frac{\gamma_j}{c_j} \frac{\delta_i}{\delta_j} + d_i^T \frac{\partial \ln T}{\partial n_i} \\
d_{ij} &= S_{ij} - \sum_{k=1}^N x_k S_{kj} \left(\frac{m_k}{m} - \frac{m_j}{m} \right) - (e_i^* - \gamma) \sum_{k=1}^N x_k S_{kj} (e_k - e_j) \\
d_i^T &= \sum_{l=1}^T - \sum_{k=1}^N c_k \sum_k^T - (e_i^* - \gamma) \sum_{k=1}^N x_k e_k \sum_k^T \\
&(i, j = 1, \dots, N)
\end{aligned} \tag{8.14'} \quad /40$$

For the case of a mixture with identical coefficients of resistance a_{ij} Expressions (8.14) and (8.14') assume, respectively, the form

$$\frac{l}{x_i} \frac{\partial x_i}{\partial n_i} = -S^* \frac{J_i}{c_i} + S^* \sum_{j=1}^N \frac{m}{m_j} \frac{J_j}{c_j} \delta_{ij} + d_i^* l \frac{\partial \ln T}{\partial n_i} \quad (8.14'')$$

$$\frac{l}{c_i} \frac{\partial c_i}{\partial n_i} = -S^* \frac{J_i}{c_i} + d_i l \frac{\partial \ln T}{\partial n_i}, \quad S^* = S_{ij} \quad (8.14''')$$

or

$$J_i = -\frac{l}{S^*} \frac{\partial c_i}{\partial n_i} + \frac{c_i}{S^*} d_i l \frac{\partial \ln T}{\partial n_i} \quad (8.14''')$$

Let us note that here we do not assume that the molecular weights of the components, their coefficients of thermal diffusion or charges are equal.

In order to give to Expressions (8.14'') the form of the Fick laws, i.e., a form similar to Expressions (8.14'''), we must introduce the concept of a molar or numerical velocity v^*

$$v^* = \sum_{k=1}^N x_k v_k$$

and molar diffusion fluxes

$$J_i' = n_i (v_i - v^*) \quad (i=1, \dots, N)$$

Then (8.14'') will be

$$\frac{l}{x_i} \frac{\partial x_i}{\partial n_i} = -S^* m \frac{J_i'}{x_i} + l \frac{\partial \ln T}{\partial n_i} d_i^*$$

If we introduce a new Schmidt number

$$S^* = m S^* = \frac{m\mu}{\rho} a_{ij} = \frac{\mu}{n} a_{ij}$$

then the expressions in (8.14'') will assume the form of the Fick laws similar to the expressions in (8.14''').

Let us return to the general case. It is natural to write Equations (8.13) in the variables

$$S = \int_0^x \rho_w \mu_w U_e r^{2k} dx', \quad n_i = n_i \quad (8.15)$$

in which they assume the simpler form

$$\delta_i^2 u_i \frac{\partial C_i}{\partial S} - (\delta_i \delta_i' f_i + \delta_i^2 \frac{\partial f_i}{\partial S}) \frac{\partial C_i}{\partial n_i} + \frac{\partial X_i}{\partial n_i} = \frac{W_i \delta_i^2}{\rho_w \mu_w \rho_w U_e r^{2k}} \quad (i=1, \dots, N) \quad (8.16)$$

At the outermost boundary of the boundary layer we shall have

$$\frac{dC_{ie}}{ds} = \frac{W_{ie}}{\rho_e} \frac{1}{\mu_w \rho_w U_e r^{2k}} \quad (i=1, \dots, N) \quad (8.17)$$

Then by allowing for (8.17) the equations in (8.16) can be rewritten as

$$\begin{aligned} \delta_i^2 u_i \frac{\partial}{\partial S} (C_i - C_{ie}) - (\delta_i \delta_i' f_i + \delta_i^2 \frac{\partial f_i}{\partial S}) \frac{\partial}{\partial n_i} (C_i - C_{ie}) + \frac{\partial X_i}{\partial n_i} = \\ = \left(\frac{W_i}{\rho U_e} - \frac{u_i W_{ie}}{\rho_e U_e} \right) \frac{\delta_i^2}{\mu_w \rho_w U_e r^{2k}} \end{aligned} \quad (8.18)$$

Let us integrate Equation (8.18) with respect to n_i from n_i to ∞ and use the asymptotic function $C_{ie}(S) - C_i(S, n_i)$, $f_i(S, n_i)$ at infinity

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$$\begin{aligned} C_e(S) - C_i(S, n_i), f_i(S, n_i) \\ u_i(S, n_i) \rightarrow 1, \frac{\partial u_i}{\partial n_i} \rightarrow 0, f_i(S, n_i) \rightarrow n_i - \psi(S), (C_e - C_i) f_i \rightarrow 0 \text{ for } n_i \rightarrow \infty \end{aligned}$$

then we find the following system of integro-differential equations

$$-J_i = \delta \delta'_i(s) [(C_e - C_i) \varphi_i + \Delta(C_i)] + \delta_i^2 \beta_i \Delta(\dot{w}_i) + \delta_i (C_e - C_i) \Psi_w \quad (8.19)$$

where

$$\Delta(C_i) = \int_{n_i}^{\infty} (C_e - C_i) u_i dn_i', \quad \beta_i = \frac{1}{\mu_w \rho_w u_e r^{2\kappa}} \quad (i=1, \dots, N) \quad (8.20)$$

$$\Delta(\dot{w}_i) = \int_{n_i}^{\infty} \left(\frac{\dot{w}_i}{\rho u_e} - \frac{\dot{w}_{ie} u_e}{\rho_e u_e} \right) dn_i'$$

If we substitute the expressions for the diffusion fluxes (8.19) into Expression (8.14) and introduce into the i -th relationship only the function ψ_i and the independent variable n_i by using the identities

$$(C_e - C_j) \left(\delta_j' \varphi_j + \delta_j \frac{\partial \varphi_j}{\partial s} \right) + \delta_j' \Delta(C_j) + \delta_j \frac{\partial}{\partial s} \Delta(C_j) = (C_e - C_j) \left(\delta_i' \varphi_i + \delta_i \frac{\partial \varphi_i}{\partial s} \right) + \delta_i' \Delta^{(i)}(C_j) + \delta_i \frac{\partial}{\partial s} \Delta^{(i)}(C_j) \quad (i, j=1, \dots, N)$$

where

$$\Delta^{(i)}(C_j) = \int_{n_i}^{\infty} (C_e - C_j^{(i)}) u_i(s, n_i') dn_i', \quad C_j^{(i)} = C_j(s, n_j) = C_j(s, \frac{\alpha_i}{\alpha_j} n_e) = C_j^{(i)}(s, n_e) \quad (i, j=1, \dots, N)$$

we then find

$$\begin{aligned} \ell \frac{\partial C_i}{\partial n_i} = & \delta_i \delta_i'(s) \sum_{j=1}^N \left\{ \frac{m}{m_j} [S_{ij} C_j (C_e - C_i) - d_{ij} C_i (C_e - C_j)] \varphi_i + \frac{m}{m_j} [S_{ij} C_j \Delta(C_i) - \right. \\ & \left. - d_{ij} C_i \Delta^{(i)}(C_j)] \right\} + \delta_i^2 \sum_{j=1}^N \left\{ \frac{m}{m_j} [S_{ij} C_j (C_e - C_i) - d_{ij} C_i (C_e - C_j)] \frac{\partial \varphi_i}{\partial s} + \frac{m}{m_j} [S_{ij} C_j (C_e - C_i) - \right. \\ & \left. - d_{ij} C_i \frac{\partial}{\partial s} \Delta^{(i)}(C_j)] \right\} + \delta_i^2 \beta_i \sum_{j=1}^N \frac{m}{m_j} [S_{ij} C_j \Delta(\dot{w}_i) - d_{ij} C_i \Delta^{(i)}(\dot{w}_j)] + \delta_i \sum_{j=1}^N \frac{m}{m_j} [S_{ij} C_j (C_e - C_i) - \\ & - d_{ij} C_i (C_e - C_j)] \Psi_w + d_{ii} C_i \ell \frac{\partial \ln T}{\partial n_i} \end{aligned} \quad (8.21)$$

where

$$\Delta^{(1)}(g) = \int_{n_i}^{\infty} (g_e - g) u_i dn_i', \quad \Delta^{(1)}(w_j) = \int_{n_i}^{\infty} \left(\frac{w_j}{\rho_e u_e} - \frac{u_i w_{ie}}{\rho_e u_e} \right) dn_i' \quad (i, j = 1, \dots, N) \quad (8.22)$$

When all the numbers S_{ij} are identical (the resistance coefficients are identical), then from (8.21) we obtain

$$-e \frac{\partial \alpha}{\partial n_i} = \delta_i \delta_i'(s) [(a_e - \alpha) \psi_i + \Delta(\alpha)] s + \delta_i^2 [(a_e - \alpha) \frac{\partial \psi_i}{\partial s} + \frac{\partial}{\partial s} \Delta(\alpha)] s + \delta_i^2 \rho_e \Delta(w_i) s + \delta_i (a_e - \alpha) \psi_i' s + \alpha \alpha' e \frac{\partial \ln T}{\partial n_i}, \quad s_{ij} = s \quad (8.23)$$

Now let us transform the diffusion equations of the elements (2.23) into independent variables

$$x' = x, \quad n_j = \frac{n}{\delta_j} \delta = \frac{z}{\delta_j} \quad (j = N+1, \dots, N) \quad (8.24)$$

and to the unknown functions

$$\begin{aligned} \gamma_j^*(x, y) &= \gamma_j^*(x, n_j) & \psi(x, y) &= \delta(x) f(x, n) = \delta_j(x) f_j(x, n_j) \\ f_j(x, n_j) &= \varphi_j(x, n_j) + f_{jw} & \varphi_j(x, n_j) &= \int_0^{n_j} u_j(x, n_j') dn_j', \quad u_j(x, n_j) = u(x, n) \\ \delta f_w &= \delta_j f_{jw} & -\rho v^k &= \delta_j f_j + \delta_j \frac{\partial f_j}{\partial x} + \delta_j u n_j x, \quad \gamma_j^* = \frac{u e^k \rho_w \mu_w \gamma_j^*(x, n_j)}{\delta_j} \end{aligned} \quad (8.25)$$

where $\delta_{N+1}(x), \dots, \delta_N(x)$ are still unknown functions. Then the equations in (2.23) will become

$$\delta_j^2 u_j \frac{\partial}{\partial x} \gamma_j^* - (\delta_j \delta_j' f_j + \delta_j^2 \frac{\partial f_j}{\partial x}) \frac{\partial \gamma_j^*}{\partial n_j} + \mu_w \rho_w u e^k \frac{\partial \gamma_j^*}{\partial n_j} = 0 \quad (j = N+1, \dots, N) \quad (8.26) \quad 142$$

In the variables

$$S = \int_0^{\infty} \rho_w \mu_w U e r^{2k} d\alpha', \quad n_j = n_j$$

the equations in (8.26) can be written more simply

$$\delta_j^2 u_j \frac{\partial q^*}{\partial s} - (\delta_j \delta_j' f_j + \delta_j^2 \frac{\partial f_j}{\partial s}) \frac{\partial q^*}{\partial n_j} + \frac{\partial \gamma_j^*}{\partial n_j} = 0 \quad (j = N_k + 1, \dots, N) \quad (8.27)$$

Let us integrate these equations with respect to n_j from n_j to ∞ and use the asymptotic formulas

$$u_j(s, n_j) \rightarrow 1, \quad \frac{\partial u}{\partial n_j} \rightarrow 0, \quad (q_e^* - q^*) f_j \rightarrow 0 \quad \text{when } n_j \rightarrow \infty$$

then we find the system of integro-differential equations

$$-\gamma_j^* = \delta_j \delta_j' [(q_e^* - q^*) \gamma_j + \Delta(q^*)] + \delta_j^2 [(q_e^* - q^*) \frac{\partial \gamma_j}{\partial s} + \frac{\partial}{\partial s} \Delta(q^*)] + \delta_j (q_e^* - q^*) \gamma_w' \quad (8.28) \\ (j = N_k + 1, \dots, N)$$

If we rewrite these equations in new independent variables

$$S = s, \quad n_* = \frac{\delta_j}{\delta_*} n_j \quad (8.29)$$

they then assume the following form

$$-\gamma_j^* \frac{\delta_j}{\delta_*} = \delta_* \delta_*' [(q_e^* - q^*) \gamma_* + \Delta^{(n)}(q^*)] + \delta_*^2 [(q_e^* - q^*) \frac{\partial \gamma_*}{\partial s} + \frac{\partial}{\partial s} \Delta^{(n)}(q^*)] + \delta_* (q_e^* - q^*) \gamma_w' \quad (8.30)$$

where

$$\Delta^*(q^*) = \int_{n_*}^{\infty} (q_e^* - q^*) u_* dn_* \quad (j = N_k + 1, \dots, N), \quad \gamma_*(s, n_*) = \int_0^{n_*} u_*(s, n_*') dn_*' \quad (8.31)$$

$$\begin{aligned} u(x, n) &= u_*(x, n_*) & \delta(x) f(x, n) &= \delta_*(x) f_*(x, n_*) \\ f_*(x, n_*) &= \varphi(x, n_*) + f_{KW} & -\rho v^k &= \delta' f_* + \delta \frac{\partial f_*}{\partial x} + \delta u n_x \end{aligned} \quad (8.32)$$

Here $\delta_*(x)$ is an arbitrary differential function of x .

Finally let us proceed to integration of the energy equation (2.42). We can first transform it into new independent variables

$$x' = x \quad n_H = \frac{2}{\delta_H(x)} = \frac{\delta}{\delta_H} n \quad (8.33)$$

and to the unknown functions.

$$\begin{aligned} H'(x', y) &= H(x, n_H) & \Psi'(x', y) &= \delta(x) f(x, n) = \delta_H(x') f_H(x, n_H) \\ \varphi_H(x, n_H) &= \int_0^{n_H} u_H(x, n_H) dn_H' & u_H(x, n_H) &= u(x, n) \\ -\rho v^k &= \delta_H' f_H + \delta_H f_H + \delta_H u n_H & \delta f_W &= \delta_H f_{HW} \\ \gamma_{iH}(x, n_H) &= \frac{\delta_H}{\delta_i} \gamma_i(x, n_i) \quad (i=1, \dots, N) & \gamma_i'(x', y) &= \frac{u_e \tau^k \mu_W \rho_W}{\delta_H} \gamma_{iH}(x, n_H) \\ \gamma_j'(x', y) &= \frac{u_e \tau^k \rho_W \mu_W}{\delta_H(x)} \gamma_{jH}(x, n_H) & (j=N+1, \dots, N) \end{aligned} \quad (8.34)$$

where $\delta_H(x)$ is still an arbitrary function. It then assumes the following form

$$\begin{aligned} \delta_H^2 u_H \frac{\partial H}{\partial x} - (\delta_H \delta_H' f_H + \delta_H^2 \frac{\partial f_H}{\partial x}) \frac{\partial H}{\partial n_H} + \sum_{j=N_H+1}^N \{ \delta_H^2 u \frac{\partial}{\partial x} [(g_e^* - g_j^*) h_j] - \\ - (\delta_H \delta_H' f_H + \delta_H^2 \frac{\partial f_H}{\partial x}) \frac{\partial}{\partial n_H} [(g_e^* - g_j) h_j] + \frac{u_e \tau^k \rho_W \mu_W}{\delta_H} \frac{\partial}{\partial n_H} (\gamma_{jH}^* h_j) \} = \end{aligned} \quad (8.35)$$

$$\begin{aligned} = u_e \tau^k \mu_W \rho_W \frac{\partial}{\partial n_H} \left\{ \frac{e}{\sigma} \left[\frac{\partial H}{\partial n_H} + (G-1) u_e^2 \frac{\partial}{\partial n_H} \left(\frac{u_H^2}{2} \right) - \sum_{j=N_H+1}^N (g_e^* - g_j^*) \frac{\partial h_j}{\partial n_H} \right] + \right. \\ \left. + \sum_{k=1}^{N_H} Q_k \left(\gamma_{kH} + \frac{e}{\sigma} \frac{\partial Q_k}{\partial n_H} \right) + R A T \sum_{j=N_H+1}^N \sum_j^T \frac{\gamma_j^*}{m_j} \right\} \end{aligned}$$

In the variables

$$s = \int_0^x \rho_w \mu_w U e^{\alpha_k} dx', \quad n_H = n_H \quad (8.36)$$

this equation will be

$$\begin{aligned} & \delta_H^2 U_H \frac{\partial H}{\partial s} - (\delta_H \delta_H' f_H + \delta_H^2 \frac{\partial f_H}{\partial s}) \frac{\partial H}{\partial n_H} + \sum_{j=N_H+1}^N \{ \delta_H U_H \frac{\partial}{\partial s} [(g_j^* - g_j^*) h_j] - \\ & - (\delta_H \delta_H' f_H + \delta_H^2 \frac{\partial f_H}{\partial s}) \frac{\partial}{\partial n_H} [(g_j^* - g_j^*) h_j] + \frac{\partial}{\partial n_H} (Y_{jH}^* h_j) \} = \frac{\partial}{\partial n_H} \left\{ \frac{e}{\sigma} \left[\frac{\partial H}{\partial n_H} + \right. \right. \quad (8.37) \\ & \left. \left. + (G-1) U e^2 \frac{\partial}{\partial n_H} \left(\frac{U_H^2}{2} \right) - \sum_{j=N_H+1}^N (g_j^* - g_j^*) \frac{\partial h_j}{\partial n_H} \right] + \sum_{k=1}^{N_H} Q_k (Y_{kH} + \frac{e}{\sigma} \frac{\partial C_k}{\partial n_H}) - R_A T \sum_{j=N_H+1}^N \sum_i^T \frac{Y_{ij}^*}{m_{ij}} \right\} \end{aligned}$$

Let us integrate Equation (8.37) by n_H from n_H to ∞ and use the asymptotic expressions

$$\frac{\partial H}{\partial n_H} \rightarrow 0, \quad (He-H) f_H \rightarrow 0 \quad \text{when } n_H \rightarrow \infty$$

Then we find the integro-differential equation of energy

$$\begin{aligned} & \frac{e}{\sigma} \left[\frac{\partial H}{\partial n_H} + (G-1) U e^2 \frac{\partial}{\partial n_H} \left(\frac{U_H^2}{2} \right) - \sum_{j=N_H+1}^N (g_j^* - g_j^*) \frac{\partial h_j}{\partial n_H} \right] + \sum_{k=1}^{N_H} Q_k (Y_{kH} + \frac{e}{\sigma} \frac{\partial C_k}{\partial n_H}) - \\ & - R_A T \sum_{j=N_H+1}^N \sum_i^T \frac{Y_{ij}^*}{m_{ij}} - \delta \delta_H' [(H-He) \Psi_H + \Delta(H)] + \delta_H^2 \left[(H-He) \frac{\partial \Psi_H}{\partial s} + \frac{\partial}{\partial s} \Delta(H) \right] + \\ & + \delta_H (H-He) \Psi_H' + C_{p2}^{(H)} \quad (8.38) \end{aligned}$$

where

$$\begin{aligned} \Delta(H) &= \int_{n_H}^{\infty} (He-H) U_H dn_H \\ C_{p2}^{(H)} &= \sum_{j=N_H+1}^N \{ \delta_H \delta_H' [(h_{je} - h_j) \Delta^{(H)}(g_j^*) - \square_j^{(H)}(g_j^*, h_j) + \delta_H^2 \left[\frac{dh_{je}}{ds} \Delta^{(H)}(g_j^*) + \right. \\ & \left. + (h_{je} - h_j) \frac{\partial}{\partial s} \Delta^{(H)}(g_j^*) - \frac{\partial}{\partial s} \square_j^{(H)}(g_j^*, h_j) \right] \} \quad (8.39) \\ \Delta^{(H)}(g_j^*) &= \int_{n_H}^{\infty} (g_j^* - g_j^*) U_H dn_H, \quad \square_j^{(H)} = \int_{n_H}^{\infty} (g_j^* - g_j^*) (h_{je} - h_j) U_H dn_H \\ & \quad (j = N_H+1, \dots, N) \end{aligned}$$

Thus, we have obtained, instead of a system of differential equations of a multi-component boundary layer with chemical reactions and ionization reactions (2.13) - (2.22), an equivalent system of integro-differential equations (8.8), (8.19), (8.21), (8.28), (8.38).

§ 9. INTEGRAL KARMAN EXPRESSIONS FOR A MULTI-COMPONENT BOUNDARY LAYER WITH REACTIONS

If in Equations (8.8), (8.19), (8.21), (8.8), (8.38) we identify all functions $\delta(S)$ with thicknesses of the respective boundary layers and set $\eta = 0$, we then obtain integral Karman expressions in terms of the variables S , n as an approximation of a boundary layer of finite thickness

$$\begin{aligned}
 \frac{\partial u(s, 0)}{\partial n} &= \delta \delta'(s) \Delta^*(u) + \delta^2 \frac{\partial}{\partial s} \Delta^*(u) + \delta^2 \beta \Delta^*(\rho) + \delta \psi_w' \\
 - \gamma_i(s, 0) &= \delta_i \delta_i' \Delta^*(c_i) + \delta_i^2 \frac{\partial}{\partial s} \Delta^*(c_i) + \delta_i^2 \beta_i \Delta^*(w_i) + \delta_i (c_{ie} - c_{iw}) \psi_w' \\
 &\quad (i=1, \dots, N) \\
 \frac{\partial c_i(s, 0)}{\partial n_i} &= \delta \delta_i'(s) \sum_{j=1}^N \frac{m_w}{m_j} [d_{ij} c_i \Delta^{*(i)}(c_j) - s_{ij} c_j \Delta^*(c_i)]_w + \\
 &\quad + \delta_i^2 \sum_{j=1}^N \frac{m_w}{m_j} [d_{ij} c_i \frac{\partial}{\partial s} \Delta^{*(i)}(c_j) - s_{ij} c_j \frac{\partial}{\partial s} \Delta^*(c_i)]_w + \delta_i^2 \sum_{l=1}^N \frac{m_w}{m_l} [d_{il} c_i (c_{le} - c_j) - s_{ij} c_j (c_{le} - c_i)] \psi_w' + \\
 &\quad + c_{iw} c_{iw} \frac{\partial \ln T}{\partial n_i}(s, 0) + \delta_i^2 \beta_i \sum_{j=1}^N \frac{m_w}{m_j} [d_{ij} c_i \Delta^{*(i)}(w_j) - s_{ij} c_j \Delta^*(w_i)]_w \quad (i=1, \dots, N) \\
 - \gamma_j^*(s, 0) &= \delta_j \delta_j' \Delta^*(g_j^*) + \delta_j^2 \frac{\partial}{\partial s} \Delta^*(g_j^*) + \delta_j (g_{je} - g_{jw}) \psi_w' \quad (j=1, \dots, N) \\
 \frac{1}{\sigma_w} \left[\frac{\partial H}{\partial n_H} - \sum_{j=N+1}^N (g_{je}^* - g_j^*) \frac{\partial h_j}{\partial n_H} \right]_w &+ \sum_{k=1}^{N_k} \alpha_{kw} (\gamma_{kh} + \frac{1}{\sigma} \frac{\partial \gamma_k}{\partial n_H})_w + R_A T_w \sum_{j=N+1}^N \sum_{l=1}^N \frac{T_j \gamma_{jl}^*}{m_j} \\
 &= \delta_H \delta_H' \Delta^*(H) + \delta_H^2 \frac{\partial}{\partial s} \Delta^*(H) + \delta_H (H_e - H_w) \psi_w' + c_p \theta_w^{(H)}
 \end{aligned} \tag{9.1}$$

where

$$\begin{aligned}
 \Delta^*(u) &= \int_0^1 u(1-u) dn \\
 \Delta^*(c_i) &= \int_0^1 (c_{ie} - c_i) u dn_i \\
 \Delta^{*(i)}(c_j) &= \int_0^1 (c_{je} - c_j) u dn_i \\
 \Delta^*(H) &= \int_0^1 (H_e - H) u dn_H \\
 &\quad (i=1, \dots, N) \\
 \Delta^*(\rho) &= \int_0^1 \left(\frac{\rho_e}{\rho} - u^2 \right) dn \\
 \Delta^*(w_i) &= \int_0^1 \left(\frac{w_i}{\rho u_e} - \frac{w_i u c_i}{\rho_e u_e} \right) dn_i \\
 \Delta^*(g_j^*) &= \int_0^1 (g_{je}^* - g_j^*) u_j dn_j \quad (j=N+1, \dots, N) \\
 \Delta^{*(l)}(w_j) &= \int_0^1 \left(\frac{w_j}{\rho u_e} - \frac{u_i w_{je}}{\rho_e u_e} \right) dn_i \\
 &\quad (j=1, \dots, N)
 \end{aligned} \tag{9.2}$$

The expressions in (9.1) are integral Karman expressions for a multi-component boundary layer with chemical reactions in the Dorodnitsyn - Stefanov variables in Lees form. On the basis of these expressions we can develop the familiar approximate method for solving a multi-component boundary layer with chemical reactions, i.e., the Karman - Pohlhausen method [4]⁽¹⁰⁾. However, in the next section, we shall develop a method for solving this problem based on use of the method of successive approximations.

§ 10. METHOD OF SUCCESSIVE APPROXIMATIONS FOR SOLVING LAMINAR MULTI-COMPONENT BOUNDARY LAYER EQUATIONS WITH CHEMICAL REACTIONS AND WITH IONIZATION REACTIONS

The problems formulated in sections 3, 4, 6, 7 for the theory of a laminar multi-component boundary layer with chemical reactions and ionization reactions are too complicated for obtaining an approximate and numerical solution in the general case. In the past these problems were solved numerically with a series of simplified assumptions, i.e., by the method of finite differences. However, this method, used in the problems studied here, leads to awkward computations that are created by the method itself rather than by the physical essence of the problem. In this section we shall develop a method of successive approximations (see § 1) for these problems. The advantage of this method consists of the simplicity for arriving at solutions using an electronic computer in the general case and in the possibility of obtaining approximate solutions (first and second approximations may be computed analytically), the accuracy of which may be estimated by this same method using an electronic computer for computing the successive approximations. Let us proceed to a discussion of this method.

We can rewrite the system of integro-differential equations (8.8),

(10) The method of Karman - Pohlhausen has as yet not been employed in the literature for solving equations of a multi-component boundary layer.

(8.19), (8.21), (8.28), (8.38) in concise form.

$$\begin{aligned}\frac{\partial u}{\partial n} &= F(x, n; u, c_1, \dots, c_N, H) \\ \frac{\partial c_i}{\partial n_i} &= \Phi_i(x, n; u, c_1, \dots, c_N, H) \quad (i=1, \dots, N)\end{aligned}\quad (10.1)$$

$$\begin{aligned}& \frac{\partial H}{\partial n_H} + (G-1) u e^2 \frac{\partial}{\partial n_H} \left(\frac{u e^2}{2} \right) - \sum_{j=N_H+1}^N (q e^* - q^*) \frac{\partial h_j}{\partial n_H} + \sum_{k=1}^{N_H} Q_k (x_{kH} G e^{-1} + \frac{\partial c_k}{\partial n_H}) + \\& + R_A T \sum_{j=N_H+1}^N \sum_j^T \frac{\gamma_j^*}{m_j} G e^{-1} = \Psi(x, n_H; u, x_1, \dots, x_N, H) \\& - \gamma_i = \delta_i \delta_i' [(c e - \alpha) \varphi_i + \Delta(\alpha)] + \delta_i^2 [(c e - \alpha) \frac{\partial \varphi_i}{\partial s} + \frac{\partial}{\partial s} \Delta(\alpha)] + \\& + \delta_i^2 \beta_1 \Delta(\dot{w}_i) + \delta_i (c e - \alpha) \psi_w' \quad (i=1, \dots, N) \\& - \gamma_j^* = \delta_j \delta_j' [(q e^* - q^*) \varphi_j + \Delta(q^*)] + \delta_j^2 [(q e^* - q^*) \frac{\partial \varphi_j}{\partial s} + \frac{\partial}{\partial s} \Delta(q^*)] + \\& + \delta_j (q e^* - q^*) \psi_w'\end{aligned}$$

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where

$$\begin{aligned}F &= \delta \delta' [(1-u) \varphi + \Delta(u)] e^{-1} + \delta^2 [(1-u) \frac{\partial \varphi}{\partial s} + \frac{\partial}{\partial s} \Delta(u)] e^{-1} + \\& + \delta^2 \beta(s) \Delta(\beta) e^{-1} + \delta (1-u) \psi_w' e^{-1} \\ \Phi_i &= \delta_i \delta_i' \sum_{j=1}^N \left\{ \frac{m}{m_j} [S_{ij} (c e - \alpha) - \alpha_{ij} \alpha (q e - q)] \varphi_i + \frac{m}{m_j} [S_{ij} c_j \Delta(\alpha) - \alpha_{ij} \Delta^{(1)}(\alpha) \alpha] \right\} e^{-1} \quad (10.2) \\& + \delta_i^2 \sum_{j=1}^N \left\{ \frac{m}{m_j} [S_{ij} q_j (c e - \alpha) - \alpha_{ij} \alpha (q e - q)] \frac{\partial \varphi_i}{\partial s} + \frac{m}{m_j} [S_{ij} c_j \frac{\partial}{\partial s} \Delta(\alpha) - \alpha_{ij} \alpha \frac{\partial}{\partial s} \Delta^{(1)}(\alpha)] \right\} e^{-1} \\& + \delta_i^2 \beta_1 \sum_{j=1}^N \frac{m}{m_j} [S_{ij} q_j \Delta(\dot{w}_i) - \alpha_{ij} \alpha \Delta^{(1)}(\dot{w}_j)] + \delta_i \sum_{j=1}^N \frac{m}{m_j} [S_{ij} q_j (c e - \alpha) - \\& - \alpha_{ij} \alpha (q e - q)] \psi_w' + \alpha_i \alpha e \frac{\partial \ln T}{\partial n_i} \\ \Psi &= \delta_H \delta_H' [(H e - H) \varphi_H + \Delta(H)] e^{-1} + \delta_H^2 [(H e - H) \frac{\partial \varphi_H}{\partial s} + \frac{\partial}{\partial s} \Delta(H)] G e^{-1} + \delta_H (H e - H) \psi_w' e^{-1} + C_{pH}^{(H)} e^{-1} G\end{aligned}$$

Let us mention that all the functions f , ψ_i and ψ have a mutually similar structure.

Let us integrate System (10.1) for the last time, respectively, with respect to the variables n , n_1 , n_2 , ..., n_N , n_H from zero to the instantaneous values of these coordinates, we then find

$$\begin{aligned} u(s, n) &= \int_0^n F dn \equiv A(s, n) \delta \delta' + [\beta B(s, n) + C(s, n)] \delta^2 + D(s, n) \Psi'_w \delta \\ C_i - C_{iw} &= \int_0^{n_i} \Phi_i dn_i \equiv A_i(s, n_i) \delta_i \delta_i' + [\beta B_i(s, n_i) + C_i(s, n_i)] \delta_i^2 + D_i(s, n_i) \Psi_w \delta_i + T_i(s, n_i) \\ H - H_w &= A(s, n_H) \delta_H \delta_H' + B_H(s, n_H) + C_H(s, n_H) \delta_H^2 + D_H(s, n_H) \Psi'_w \delta_H \end{aligned} \quad (10.3)$$

where

$$\begin{aligned} A(s, n) &= \int_0^n [(1-u)\varphi + \Delta(u)] e^{-1} dn, \quad B(s, n) = \int_0^n \Delta(s) e^{-1} dn \\ C(s, n) &= \int_0^n \left[(1-u) \frac{\partial \varphi}{\partial s} + \frac{\partial}{\partial s} \Delta(u) \right] e^{-1} dn, \quad D(s, n) = \int_0^n (1-u) e^{-1} dn \\ A_i(s, n_i) &= \sum_{j=1}^N a_{ij}, \quad B_i(s, n_i) = \sum_{j=1}^N b_{ij}, \quad C_i(s, n_i) = \sum_{j=1}^N c_{ij}(s, n_i), \\ D_i(s, n_i) &= \sum_{j=1}^N d_{ij}(s, n_i) \\ a_{ij} &= \int_0^{n_i} \left\{ \frac{m}{m_j} [S_{ij} g_j (a_e - a_i) - d_{ij} a_i (g_e - g_j)] \varphi_i + \frac{m}{m_j} [S_{ij} g_j \Delta(a_i) - d_{ij} a_i \Delta^{(i)}(g_j)] \right\} e^{-1} dn_i \\ b_{ij} &= \int_0^{n_i} \frac{m}{m_j} [S_{ij} g_j (a_e - a_i) - d_{ij} a_i (g_e - g_j)] e^{-1} dn_i \\ T_i &= \int_0^{n_i} d_i a_i \frac{\partial \ln T}{\partial n_i} dn_i c_{ij} = \int_0^{n_i} \left\{ \frac{m}{m_j} [S_{ij} g_j (g_e - g_j) - d_{ij} a_i (g_e - g_j)] \frac{\partial \varphi_i}{\partial s} + \frac{m}{m_j} [S_{ij} g_j \frac{\partial}{\partial s} \Delta(a_i) - d_{ij} a_i \frac{\partial}{\partial s} \Delta^{(i)}(g_j)] \right\} e^{-1} dn_i \end{aligned} \quad (10.4)$$

$$\begin{aligned}
 d_{ij} &= \int_0^{n_H} \frac{u_i}{u_j} [s_{ij} C_j (C_e - C_i) - d_{ij} C_i (C_e - C_i)] e^{-1} dn_H \\
 A(s, n_H) &= \int_0^{n_H} [(H_e - H) \varphi_H + \Delta(H)] G e^{-1} dn_H \\
 B(s, n_H) &= \int_0^{n_H} \left[-(1-\sigma) U e^2 \frac{\partial}{\partial n_H} \left(\frac{U_H^2}{2} \right) + \sum_{j=N_H+1}^N (C_j^* - C_j^*) \frac{\partial h_j}{\partial n_H} - \sum_{K=1}^{N_H} Q_K (\gamma_{KH} G e^{-1} \right. \\
 &\quad \left. + \frac{\partial Q_K}{\partial n_H}) - R_A T \sum_{j=N_H+1}^N \sum_j^T \frac{\gamma_j^*}{m_j} G e^{-1} + c_p^{(H)} G e^{-1} \right] dn_H \\
 C(s, n_H) &= \int_0^{n_H} [(H_e - H) \frac{\partial \varphi_H}{\partial s} + \frac{\partial}{\partial s} \Delta(H)] G e^{-1} dn_H \\
 D(s, n_H) &= \int_0^{n_H} (H_e - H) G e^{-1} dn_H
 \end{aligned}$$

System (10.3) represents a system of integro-differential equations. Now let us select the functions $\delta(s)$, $\delta_1(s)$, ..., $\delta_N(s)$, $\delta_H(s)$, which were previously arbitrary, in such a manner that after substitution into the right-hand sides of System (10.3) of the arbitrary and integrable functions $u(s, n)$, $(C_i(s, n_i))$ ($i = 1, \dots, N$), $H(s, n_H)$, which satisfy only the boundary conditions on the wall and at infinity, we obtain from left to right: 1, $C_{ie} - C_{iw}$ ($i = 1, \dots, N$) and $H_e - H_w$, respectively. Then the functions δ for the specified functions u , C_i ($i = 1, \dots, N$), H which satisfy the boundary conditions will be found from solving the system of ordinary differential equations

$$\begin{aligned}
 1 &= A(s, \infty) \delta \delta' + [\beta(s) B(s, \infty) + C(s, \infty)] \delta^2 + D(s, \infty) \psi_w' \delta \\
 C_{ie} - C_{iw} &= A_i(s, \infty) \delta_i \delta_i' + [\beta_i B_i(s, \infty) + C_i(s, \infty)] \delta_i^2 + D_i(s, \infty) \psi_w' \delta_i + T_i(s, \infty) \\
 H_e - H_w &= A_H(s, \infty) \delta_H \delta_H' + B_H(s, \infty) + C_H(s, \infty) \delta_H^2 + D_H(s, \infty) \psi_w' \delta_H
 \end{aligned} \tag{10.5}$$

Now the process of successive approximations will be constructed in the following manner. From the practical data on the qualitative character

of the profiles of the unknown functions in the boundary layer let us select the zero approximation:

$$u(s, n) = u^{(0)}(s, n), \quad c_i(s, n_i) = c_i^{(0)}(s, n_i) \quad (i=1, \dots, N), \quad H(s, n_H) = H^{(0)}(s, n_H).$$

Then we solve the system of consistent ordinary differential equations (10.5) with the initial equations⁽¹¹⁾

$$\delta(0) = \delta_L(0) = \dots = \delta_N(0) = \delta_H(0) = 0 \quad (10.6)$$

Then from (10.3) we find $u^{(1)}(s, n)$, $c_i^{(1)}(s, n_i)$ ($i=1, \dots, N$), $H^{(1)}(s, n_H)$, and then the process is repeated until the functions $u(s, \eta)$, $c_i(s, \eta)$, ($i=1, \dots, N$), $H(s, \eta)$ in the physical variables are no longer near, respectively, to one another in the above given algorithm

$$\frac{1}{2} \frac{d}{ds} (\delta^{(k)})^2 + \left[\beta(s) \frac{B^{(k)}(s, \infty)}{A^{(k)}(s, \infty)} + \frac{C^{(k)}(s, \infty)}{A^{(k)}(s, \infty)} \right] (\delta^{(k)})^2 + \frac{D^{(k)}(s, \infty)}{A^{(k)}(s, \infty)} \psi_w' \delta^{(k)} = \frac{1}{A^{(k)}(s, \infty)}$$

$$\frac{1}{2} \frac{d}{ds} (\delta_i^{(k)})^2 + \left[\beta_i(s) \frac{B_i^{(k)}(s, \infty)}{A_i^{(k)}(s, \infty)} + \frac{C_i^{(k)}(s, \infty)}{A_i^{(k)}(s, \infty)} \right] (\delta_i^{(k)})^2 + \frac{D_i^{(k)}(s, \infty)}{A_i^{(k)}(s, \infty)} \psi_w' \delta_i^{(k)} + \frac{T_i^{(k)}(s, \infty)}{A_i^{(k)}(s, \infty)} = \frac{C_{ie} - C_{iw}}{A_i^{(k)}(s, \infty)}$$

$$\frac{1}{2} \frac{d}{ds} (\delta_H^{(k)})^2 + \frac{C_H^{(k)}(s, \infty)}{A_H^{(k)}(s, \infty)} (\delta_H^{(k)})^2 + \frac{D_H^{(k)}(s, \infty)}{A_H^{(k)}(s, \infty)} \psi_w' \delta_H^{(k)} = \frac{H_e - H_w - B_H^{(k)}(s, \infty)}{A_H^{(k)}(s, \infty)}$$

Equation continues on next page

(11) Let us note that this initial condition determines the only finite solution to System (10.5) when $0 \leq x < X$, where X depends on the data in the problem.

$$\begin{aligned}
u^{(k+1)}(s, n) &= \frac{A^{(k)}(s, n)}{A^{(k)}(s, \infty)} + \left\{ \beta(s) \left[B^{(k)}(s, n) - \frac{A^{(k)}(s, n)}{A^{(k)}(s, \infty)} B^{(k)}(s, \infty) \right] + C^{(k)}(s, n) - \right. \\
&\quad \left. - \frac{A^{(k)}(s, n)}{A^{(k)}(s, \infty)} C^{(k)}(s, \infty) \right\} (\delta^{(k)})^2 + \left[D^{(k)}(s, n) - \frac{A^{(k)}(s, n)}{A^{(k)}(s, \infty)} D^{(k)}(s, \infty) \right] \psi_w' \delta^{(k)} \\
C_i^{(k+1)} - C_{iw} &= (C_{ie} - C_{iw}) \frac{A_i^{(k)}(s, n_i)}{A_i^{(k)}(s, \infty)} + \left\{ \beta_L(s) \left[B_i^{(k)}(s, n_i) - \frac{A_i^{(k)}(s, n_i)}{A_i^{(k)}(s, \infty)} B_i^{(k)}(s, \infty) \right] + \right. \\
&\quad \left. + C_i^{(k)}(s, n_i) - \frac{A_i^{(k)}(s, n_i)}{A_i^{(k)}(s, \infty)} C_i^{(k)}(s, \infty) \right\} (\delta_i^{(k)})^2 + \left[D_i^{(k)}(s, n_i) - \frac{A_i^{(k)}(s, n_i)}{A_i^{(k)}(s, \infty)} D_i^{(k)}(s, \infty) \right] \psi_w' \delta_i^{(k)} + \\
&\quad + T_i^{(k)}(s, n_i) - \frac{A_i^{(k)}(s, n_i)}{A_i^{(k)}(s, \infty)} T_i^{(k)}(s, \infty)
\end{aligned} \tag{10.7}$$

(con't.)

$$\begin{aligned}
H^{(k+1)} - H_w &= (H_e - H_w) \frac{A_H^{(k)}(s, n_H)}{A_H^{(k)}(s, \infty)} + B_H^{(k)}(s, n_H) - \frac{A_H^{(k)}(s, n_H)}{A_H^{(k)}(s, \infty)} B_H^{(k)}(s, \infty) + \\
&\quad + \left[C_H^{(k)}(s, n_H) - \frac{A_H^{(k)}(s, n_H)}{A_H^{(k)}(s, \infty)} C_H^{(k)}(s, \infty) \right] (\delta_H^{(k)})^2 + \left[D_H^{(k)}(s, n_H) - \right. \\
&\quad \left. - \frac{A_H^{(k)}(s, n_H)}{A_H^{(k)}(s, \infty)} D_H^{(k)}(s, \infty) \right] \psi_w' \delta_H^{(k)}
\end{aligned} \tag{10.8}$$

When $\psi_w' \neq 0$, the equations in (10.7) represent a system of consistent differential Abel equations of the second kind, not integrable by quadratures.

In solving the problems in boundary layer theory on an impermeable wall, when $\psi_w' = 0$, System (10.7) may be integrated. Here if we allow for the conditions in (10.6), we find

$$\begin{aligned}
(\delta(s))^{(k)2} &= \exp\left(-2 \int_0^s y^{(k)}(s') ds'\right) \int_0^s \frac{2}{A(s', \infty)} \exp\left(2 \int_0^{s'} y^{(k)}(s'') ds''\right) ds' \\
y^{(k)}(s) &= \beta(s) \frac{B^{(k)}(s, \infty)}{A^{(k)}(s, \infty)} + \frac{C^{(k)}(s, \infty)}{A^{(k)}(s, \infty)} \\
\delta_i^{(k)2} &= \exp\left(-2 \int_0^s y_i^{(k)}(s') ds'\right) \int_0^s \frac{2 [C_{ie} - C_{iw} - T_i^{(k)}(s', \infty)]}{A_i^{(k)}(s', \infty)} \exp\left(2 \int_0^{s'} y_i^{(k)}(s'') ds''\right) ds' \\
y_i^{(k)}(s) &= \beta_i(s) \frac{B_i^{(k)}(s, \infty)}{A_i^{(k)}(s, \infty)} + \frac{C_i^{(k)}(s, \infty)}{A_i^{(k)}(s, \infty)} \quad (i=1, \dots, N) \quad (10.9) \\
\delta_H^{(k)2} &= \exp\left(-2 \int_0^s y_H^{(k)}(s') ds'\right) \int_0^s \frac{2 [H_e - H_w - B_H^{(k)}(s', \infty)]}{A_H^{(k)}(s', \infty)} \exp\left(2 \int_0^{s'} y_H^{(k)}(s'') ds''\right) ds' \\
y_H^{(k)}(s) &= \frac{C_H^{(k)}(s, \infty)}{A_H^{(k)}(s, \infty)}
\end{aligned}$$

or in the physical variable x ($ds = \mu_w \rho_w l e r^{2k} dx$)

$$\begin{aligned}
\delta^{(k)2}(x) &= \exp\left(-2 \int_0^x y^{(k)}(x') dx'\right) \int_0^x \frac{2 \mu_w \rho_w l e r^{2k}}{A(x', \infty)} \exp\left(2 \int_0^{x'} y^{(k)}(x'') dx''\right) dx' \\
y^{(k)}(x) &= \beta(x) \frac{B^{(k)}(x, \infty)}{A^{(k)}(x, \infty)} + \frac{C^{(k)}(x, \infty)}{A^{(k)}(x, \infty)}, \quad \beta(x) = \frac{d \ln l e}{dx} \\
\delta_i^{(k)2}(x) &= \exp\left(-2 \int_0^x y_i^{(k)}(x') dx'\right) \int_0^x \frac{2 [C_{ie} - C_{iw} - T_i^{(k)}(x', \infty)]}{A_i^{(k)}(x', \infty)} \mu_w \rho_w l e r^{2k} \exp\left(2 \int_0^{x'} y_i^{(k)}(x'') dx''\right) dx' \\
y_i^{(k)}(x) &= \beta_i(x) \frac{B_i^{(k)}(x, \infty)}{A_i^{(k)}(x, \infty)} + \frac{C_i^{(k)}(x, \infty)}{A_i^{(k)}(x, \infty)} \quad (i=1, \dots, N) \quad (10.10) \quad /48 \\
\delta_H^{(k)2}(x) &= \exp\left(-2 \int_0^x y_H^{(k)}(x') dx'\right) \int_0^x \frac{2 [H_e - H_w - B_H^{(k)}(x', \infty)]}{A_H^{(k)}(x', \infty)} \mu_w \rho_w l e r^{2k} \exp\left(2 \int_0^{x'} y_H^{(k)}(x'') dx''\right) dx' \\
y_H^{(k)}(x) &= \frac{C_H^{(k)}(x, \infty)}{A_H^{(k)}(x, \infty)}
\end{aligned}$$

If we substitute the solution to (10.9) into (10.8), we find an explicit closed scheme for the method of successive approximations when $\psi_w' = 0$

$$\begin{aligned}
u^{(k+1)}(s, n) &= \frac{A^{(k)}(s, n)}{A^{(k)}(s, \infty)} + \left\{ \beta(s) \left[B^{(k)}(s, n) - \frac{A^{(k)}(s, n)}{A^{(k)}(s, \infty)} B^{(k)}(s, \infty) \right] + C^{(k)}(s, n) - \right. \\
&- \frac{A^{(k)}(s, n)}{A^{(k)}(s, \infty)} C^{(k)}(s, \infty) \left. \right\} \exp \left(-2 \int_0^s y^{(k)}(s') ds' \right) \int_0^s \frac{2}{A^{(k)}(s', \infty)} \exp \left(2 \int_0^{s'} y^{(k)}(s'') ds'' \right) ds' \\
C_i^{(k+1)}(s, n_i) - C_{iw}(s) &= (C_{ie} - C_{iw}) \frac{A_i^{(k)}(s, n_i)}{A_i^{(k)}(s, \infty)} + \left\{ \beta_i(s) \left[B_i^{(k)}(s, n_i) - \right. \right. \\
&- \frac{A_i^{(k)}(s, n_i)}{A_i^{(k)}(s, \infty)} B_i^{(k)}(s, \infty) \left. \right] + C_i^{(k)}(s, n_i) - \\
&- \frac{A_i^{(k)}(s, n_i)}{A_i^{(k)}(s, \infty)} C_i^{(k)}(s, \infty) \left. \right\} \exp \left(-2 \int_0^s y_i^{(k)}(s') ds' \right) \int_0^s \frac{2(C_{ie} - C_{iw} - T_i(s, \infty))}{A_i^{(k)}(s', \infty)} \exp \left(2 \int_0^{s'} y_i^{(k)}(s'') ds'' \right) ds' \\
H^{(k+1)}(s, n_H) - H_w(s) &= (H_e - H_w) \frac{A_H^{(k)}(s, n_H)}{A_H^{(k)}(s, \infty)} + B_H^{(k)}(s, n_H) - \frac{A_H^{(k)}(s, n_H)}{A_H^{(k)}(s, \infty)} B_H^{(k)}(s, \infty) + \left[C_H^{(k)}(s, n_H) - \right. \\
&- \frac{A_H^{(k)}(s, n_H)}{A_H^{(k)}(s, \infty)} C_H^{(k)}(s, \infty) \left. \right] \exp \left(-2 \int_0^s y_H^{(k)}(s') ds' \right) \int_0^s \frac{2(H_e - H_w - B_H^{(k)}(s', \infty))}{A_H^{(k)}(s', \infty)} \exp \left(2 \int_0^{s'} y_H^{(k)}(s'') ds'' \right) ds'
\end{aligned} \tag{10.11}$$

or in the physical variable x

$$\begin{aligned}
u^{(k+1)}(x, n) &= \frac{A^{(k)}(x, n)}{A^{(k)}(x, \infty)} + \left\{ \beta(x) \left[B^{(k)}(x, n) - \frac{A^{(k)}(x, n)}{A^{(k)}(x, \infty)} B^{(k)}(x, \infty) \right] + \right. \\
&+ C^{(k)}(x, n) - \frac{A^{(k)}(x, n)}{A^{(k)}(x, \infty)} C^{(k)}(x, \infty) \left. \right\} \frac{1}{\mu_w \beta_w U e^{2k}} \exp \left(-2 \int_0^x y^{(k)}(x') dx' \right) \int_0^x \frac{2 \mu_w \beta_w U e^{2k}}{A^{(k)}(x', \infty)} \exp \left(2 \int_0^{x'} y^{(k)}(x'') dx'' \right) dx' \\
C_i^{(k)}(x, n_i) - C_{iw}(x) &= (C_{ie} - C_{iw}) \frac{A_i^{(k)}(x, n_i)}{A_i^{(k)}(x, \infty)} + \left\{ \beta_i(x) \left[B_i^{(k)}(x, n_i) - \frac{A_i^{(k)}(x, n_i)}{A_i^{(k)}(x, \infty)} B_i^{(k)}(x, \infty) \right] + C_i^{(k)}(x, n_i) - \right. \\
&- \frac{A_i^{(k)}(x, n_i)}{A_i^{(k)}(x, \infty)} C_i^{(k)}(x, \infty) \left. \right\} \frac{1}{\mu_w \beta_w U e^{2k}} \exp \left(-2 \int_0^x y_i^{(k)}(x') dx' \right) \int_0^x \frac{2(C_{ie} - C_{iw} - T_i(x', \infty)) \mu_w \beta_w U e^{2k}}{A_i^{(k)}(x', \infty)} \exp \left(2 \int_0^{x'} y_i^{(k)}(x'') dx'' \right) dx' \\
H^{(k+1)}(x, n_H) - H_w(x) &= (H_e - H_w) \frac{A_H^{(k)}(x, n_H)}{A_H^{(k)}(x, \infty)} + B_H^{(k)}(x, n_H) - \frac{A_H^{(k)}(x, n_H)}{A_H^{(k)}(x, \infty)} B_H^{(k)}(x, \infty) + \\
&+ \left[C_H^{(k)}(x, n_H) - \frac{A_H^{(k)}(x, n_H)}{A_H^{(k)}(x, \infty)} C_H^{(k)}(x, \infty) \right] \frac{1}{\mu_w \beta_w U e^{2k}} \exp \left(-2 \int_0^x y_H^{(k)}(x') dx' \right) \int_0^x \frac{2[H_e - H_w - B_H^{(k)}(x', \infty)]}{A_H^{(k)}(x', \infty)} \exp \left(2 \int_0^{x'} y_H^{(k)}(x'') dx'' \right) dx'
\end{aligned} \tag{10.12}$$

After carrying out the necessary number of iterations, determined by the

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specified accuracy of the obtained solution, we can compute the local friction on the wall using (8.8)

$$\tau_w(s) \tau^k = \left(\mu \frac{\partial u'}{\partial y'} \right)_w \tau^k = \frac{\mu_w \rho_w U_e \tau^{2k}}{\delta(s)} \frac{\partial u(s, 0)}{\partial n} = \mu_w \rho_w U_e \tau^{2k} \left\{ \frac{\Delta^*(u)}{A(s, \infty) \delta(s)} + \left[\beta(s) \left(\Delta^*(\rho) - \Delta^*(u) \frac{B(s, \infty)}{A(s, \infty)} \right) + \frac{\partial}{\partial s} \Delta^*(u) - \Delta^*(u) \frac{C(s, \infty)}{A(s, \infty)} \right] \delta(s) + \left[1 - \Delta^*(u) \frac{D(s, \infty)}{A(s, \infty)} \right] \psi_w' \right\} \quad (10.13)$$

where

$$\Delta^*(u) = \int_0^\infty u(1-u) dn', \quad \Delta^*(\rho) = \int_0^\infty \left(\frac{\rho_e}{\rho} - u^2 \right) dn' \quad (10.14)$$

or in the physical variable x

$$\frac{\tau_w(x) \tau^k}{U_e} = \frac{\mu_w \rho_w U_e \tau^{2k} \Delta^*(u)}{A(x, \infty) \delta(x)} + \left\{ \beta(x) \left[\Delta^*(\rho) - \Delta^*(u) \frac{B(x, \infty)}{A(x, \infty)} \right] + \frac{\partial}{\partial x} \Delta^*(u) - \Delta^*(u) \frac{C(x, \infty)}{A(x, \infty)} \right\} \delta(x) - \left[1 - \Delta^*(u) \frac{D(x, \infty)}{A(x, \infty)} \right] (\rho \nu' \tau^k)_w \quad (10.15)$$

The diffusion fluxes of the components on the wall will be, according to (8.12) and (8.19),

$$- \mathcal{J}_i'(s, 0) \tau^k = \frac{\mu_w \rho_w U_e \tau^{2k}}{\delta_i(s)} \left\{ \frac{\Delta^*(\alpha_i)}{A_i(s, \infty)} [C_{ie} - C_{iw} - T_i(s, \infty)] + \left[\beta_i(s) \left(\Delta^*(w_i) - \Delta^*(\alpha_i) \frac{B_i(s, \infty)}{A_i(s, \infty)} \right) + \frac{\partial}{\partial s} \Delta^*(\alpha_i) - \Delta^*(\alpha_i) \frac{C_i(s, \infty)}{A_i(s, \infty)} \right] \delta_i^2(s) + [C_{ie} - C_{iw} - \Delta^*(\alpha_i) \frac{D_i(s, \infty)}{A_i(s, \infty)}] \psi_w' \delta_i(s) \right\} \quad (10.16)$$

($i=1, \dots, N$)

where

$$\Delta^*(\alpha_i) = \int_0^\infty (C_{ie} - C_i) u_i dn', \quad \Delta^*(w_i) = \int_0^\infty \left(\frac{w_i}{\rho_e U_e} - \frac{u_i w_i C_{ie}}{\rho_e U_e} \right) dn' \quad (10.17)$$

or in the physical variable x

$$\begin{aligned}
 -\gamma'_i(x,0) \tau^k = & \left\{ \frac{\mu_w \rho_w U_e \tau^{*k} \Delta^*(\alpha)}{A_i(x,\infty) \delta_i(x)} [C_{ie} - C_{iw} - T_i(s,\infty)] \right\} + [\beta_L(x) (\Delta^*(w_i) - \\
 & - \Delta^*(\alpha) \frac{B_i(x,\infty)}{A_i(x,\infty)}) + \frac{\partial}{\partial x} \Delta^*(\alpha) - \Delta^*(\alpha) \frac{C_i(x,\infty)}{A_i(x,\infty)}] \delta_i(x) - \\
 & - [C_{ie} - C_{iw} - \Delta^*(\alpha) \frac{D_i(x,\infty)}{A_i(x,\infty)}] (\rho \nu \tau^k)_w
 \end{aligned} \quad (10.18)$$

The diffusion fluxes of the elements on the wall, using their definitions in (2.23), will be

$$\gamma_j^*(x,0) = \gamma'_j(x,0) + \sum_{k=1}^M \frac{\nu_{kj} m_j}{m_k} \gamma_k'(x,0) \quad (10.19)$$

The concentration gradients on the wall can be found from expressions in (8.21), which will be, after excluding Expressions δ_i δ_i' using the Equations in (10.7)

$$\begin{aligned}
 \frac{1}{\rho_w U_e \tau^k} \frac{\partial C_i(s,0)}{\partial y} = \frac{1}{\delta_i} \frac{\partial C_i(s,0)}{\partial \eta_i} = \frac{(C_{ie} - C_{iw} - T_i(s,\infty))}{A_i(s,\infty) \delta_i(s)} \sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j \Delta^*(\alpha) - \\
 - \alpha_{ij} \alpha \Delta^{*(i)}(g_j))_w + \left\{ \beta_L(s) \left[\sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j \Delta^*(w_i) - \alpha_{ij} \alpha \Delta^{*(i)}(w_j))_w - \right. \right. \\
 \left. \left. - \frac{B_i(s,\infty)}{A_i(s,\infty)} \sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j \Delta^*(\alpha) - \alpha_{ij} \alpha \Delta^{*(i)}(g_j))_w \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
 + \sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j \frac{\partial}{\partial s} \Delta^*(\alpha) - \alpha_{ij} \alpha \frac{\partial}{\partial s} \Delta^{*(i)}(g_j))_w - \frac{C_i(s,\infty)}{A_i(s,\infty)} \sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j \Delta^*(\alpha) - \\
 - \alpha_{ij} \alpha \Delta^{*(i)}(g_j))_w \} \delta_i + \left[\sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j (C_{ie} - C_{iw}) - \alpha_{ij} \alpha (C_{je} - g_j w))_w - \right. \\
 \left. - \frac{D_i(s,\infty)}{A_i(s,\infty)} \sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j \Delta^*(\alpha) - \alpha_{ij} \alpha \Delta^{*(i)}(g_j))_w \right] \gamma_w' + \frac{C_{iw} C_{iw}}{\delta_i(s)} \frac{\partial \ln T(s,0)}{\partial \eta_i} \quad (10.20) \quad /50
 \end{aligned}$$

or in the physical variable x

$$\begin{aligned}
 \mu_w \tau^k \frac{\partial \alpha(x, 0)}{\partial y} &= \frac{\mu_w p_w U_e \tau^{2k}}{\delta_i(x)} \frac{\partial \alpha(x, 0)}{\partial \eta_i} = \frac{\mu_w p_w U_e \tau^{2k}}{A_i(x, \infty) \delta_i(x)} (\alpha_e - \alpha_w - T_i(x, \infty)). \\
 \cdot \sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j \Delta^*(\alpha_i) - d_{ij} \alpha_i \Delta^*(g_j))_w &+ \left\{ \beta_1(x) \left[\sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j \Delta^*(\alpha_i) - d_{ij} \alpha_i \Delta^*(g_j))_w \right. \right. \\
 - \frac{B_i(x, \infty)}{A_i(x, \infty)} \sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j \Delta^*(\alpha_i) - d_{ij} \alpha_i \Delta^*(g_j))_w &+ \sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j \frac{\partial}{\partial x} \Delta^*(\alpha_i) - \\
 - d_{ij} \alpha_i \frac{\partial}{\partial x} \Delta^*(g_j))_w - \frac{\alpha_i(x, \infty)}{A_i(x, \infty)} \sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j \Delta^*(\alpha_i) - d_{ij} \alpha_i \Delta^*(g_j))_w &\left. \right\} \delta_i(x) - \\
 - \left[\sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j (\alpha_e - \alpha_w) - d_{ij} \alpha_i (g_e - g_w))_w - \frac{D_i(x, \infty)}{A_i(x, \infty)} \sum_{j=1}^N \frac{m_w}{m_j} (S_{ij} g_j \Delta^*(\alpha_i) - \right. \\
 \left. - d_{ij} \alpha_i \Delta^*(g_j))_w \right] (\rho \tau^k)_w &+ \mu_w \tau^k d_w \alpha_w \frac{\partial \alpha}{\partial y}
 \end{aligned} \tag{10.21}$$

The enthalpy gradient (temperature) on the wall, using (8.38) and (10.7), will be

$$\begin{aligned}
 \frac{1}{\sigma} \left[\frac{\partial H(s, 0)}{\partial n_H} - \sum_{j=N_k+1}^N (g_e^* - g_j^*) \frac{\partial h_j(s, 0)}{\partial n_H} \right] &+ \sum_{k=1}^{N_k} Q_{kw} \left(\gamma_{KH} + \frac{1}{\sigma} \frac{\partial \alpha_k}{\partial n_H} \right)_w + \\
 + R_A T_w \sum_{j=N_k+1}^N \sum_j^T \frac{g_{jw}^*}{m_j} &= \frac{\Delta^*(H)}{A_H(s, \infty)} [H_e - H_w - B_H(s, \infty)] + \\
 + \left[\frac{\partial}{\partial s} \Delta^*(H) - \Delta^*(H) \frac{C_H(s, \infty)}{A_H(s, \infty)} \right] \delta_H^2(s) &+ [H_e - H_w - \Delta^*(H) \frac{D_H(s, \infty)}{A_H(s, \infty)}] \gamma_w' \delta_H(s) + \\
 + C_{paw}^{(H)} &
 \end{aligned} \tag{10.22}$$

or in the variable x

$$\begin{aligned} & \frac{1}{\sigma} \left[\frac{\partial H(x, 0)}{\partial n_H} - \sum_{j=N_k+1}^N (c_j^* - q_j) \frac{\partial h_j(x, 0)}{\partial n_H} \right] + \sum_{k=1}^{N_k} Q_{kw} \left(Y_{kH} + \frac{1}{\sigma} \frac{\partial c_k}{\partial n_H} \right)_w + \\ & + R_A T_w \sum_{j=N_k+1}^N \sum_j^T \frac{Y_{jw}^*}{m_j} = \frac{\delta_H(x)}{\mu_w \rho_w U_e r^{2k}} \left\{ \frac{\mu_w \rho_w U_e r^{2k} \Delta^*(H)}{A_H(x, \infty) \delta_H(x)} [H_e - H_w - B_H(x, \infty)] + \right. \\ & + \left[\frac{\partial}{\partial x} \Delta^*(H) - \Delta^*(H) \frac{C_H(x, \infty)}{A_H(x, \infty)} \right] \delta_H(x) + [H_e - H_w - \Delta^*(H) \frac{\partial H(x, \infty)}{A_H(x, \infty)}] \frac{\partial H(x, \infty)}{\delta_H(x)} \Bigg\} + \\ & + C_{p\delta w}^{(H)} \frac{\mu_w \rho_w U_e r^{2k}}{\delta_H(x)} \end{aligned} \quad (10.23)$$

In solving problems in boundary layer theory on a nondisintegrating wall, the solution to the problem is finished⁽¹²⁾ for computing the friction and the heat flux toward the wall with a specified temperature T_w .

In solving problems in boundary layer theory over a thermochemically disintegrating wall, the expression obtained for the diffusion fluxes of the components and elements as well as the enthalpy gradient must be substituted at each step of the iterations into the respective boundary conditions at the disintegration front after which a system of nonlinear integral Volterra equations is obtained for determining the concentrations and temperature on the wall, as well as the mass transport velocity.

CONCLUSIONS

1. We have given a strict formulation of the problems in laminar boundary layer theory with chemical reactions and ionization reactions both on nondisintegrating and on thermochemically disintegrating surfaces, made of a material having a complex chemical composition. Special attention is devoted to a rational formulation of these problems. We have discussed in detail the formulation of the problem on chemically balanced flows in boundary layers for mixtures of gases with different diffusion properties of the com-

(12) If we do not examine the problem concerning the thermometer.

ponents. We have shown that even in the case of a nondisintegrating surface, when no new elementary chemical composition is added to the flow, in a chemically balanced boundary layer for a mixture with different diffusion properties of the components the elementary chemical composition at each point of the boundary layer is not equal to the elementary composition in the approaching flow. Therefore, computation of the transport coefficients according to the previously given tabular data on the balanced composition of the approaching flow in the general case is not exact and may lead to additional errors.

2. We have given a new method for obtaining "exact" numerical solutions to arbitrary two-dimensional problems in boundary layer theory, based on use of the method of successive approximations. We have given a method for obtaining analytical approximate solutions to the problems in boundary layer theory, which are the first approximations of this method. The accuracy may be evaluated any time using the computations of the next approximations on an electronic computer. For a reasonable choice of the zero approximation even the third approximation will give a high accuracy to the solution.

3. The proposed method may be expanded to solving nonstationary two-dimensional, stationary three-dimensional and nonstationary three-dimensional problems in boundary layer theory.

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